

PERT

Computerized Placement Test

REVIEW BOOKLET FOR MATHEMATICS

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Orlando, Florida*

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PERT Review for Mathematics

This PERT Review was created to help students **to review** the major skills that are assessed on the PERT test in order to achieve the most accurate placement into a course of mathematics at Valencia College.

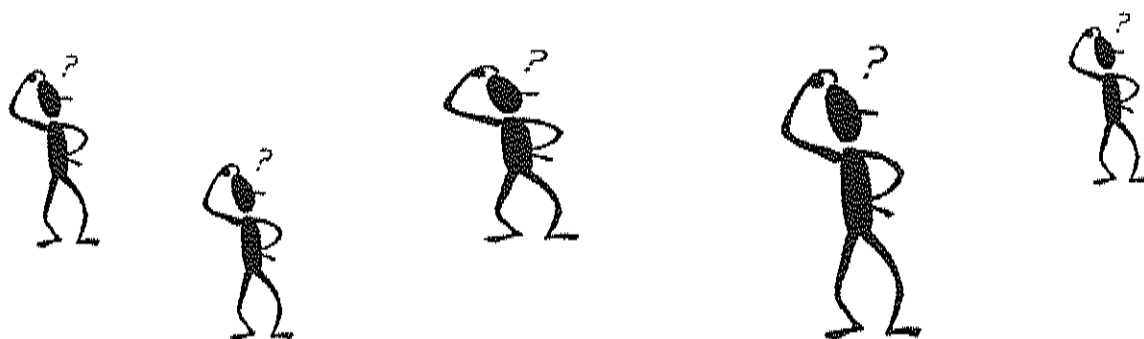
If you have not learned the subject matter covered in this booklet at an earlier time, it is unlikely that you will be able to learn it for the first time through this review.

No calculator will be allowed on the PERT test.

The PERT test does NOT have a time limit.

Math Study Skills Tip Sheet

1. Read your textbook.
 - Read your textbook before the topic is covered in class. Make notes on anything you do not understand, so you can get that cleared up during class.
 - It is crucial to master one concept before going on to the next and to stay current with your reading.
 - Read actively - read with paper and pencil in hand - work out examples yourself - highlight and take notes.
2. Set up a regular study time and place.
 - Be aware of your best time of day to study.
 - Study two hours for every hour you spend in class.
 - Because most math classes are cumulative, it is better to study for a shorter amount of time more often, then to wait until the day before the test.
 - Study math first if it is your most difficult subject.
 - Study with others - you will learn different approaches to reaching solutions.
3. Be actively involved.
 - Attend class regularly.
 - Come to class with homework completed, if possible.
 - Speak up when you have a question.
 - Seek extra help, if necessary.



Math Centers with free tutors are available
on all campuses of Valencia College.

Part 1

General Mathematics

All students entering Valencia College should review this section.

Some of this general mathematics material will be taught in the Developmental Math I course.

For any math class higher than Developmental Math I the instructor will assume their students know everything in this section.

NOTE: The PERT test will not specifically test anything in this section but questions on the test will assume you know your basics in order to answer some of the algebra questions.

Test for General Mathematics

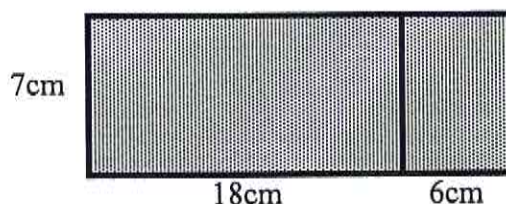
Choose the one alternative that best completes the statement or answers the question.

Evaluate:

- | | | | | |
|-----------------------------|--------|--------|--------|--------|
| 1) $19 + 13 \cdot 16$ | a) 263 | b) 512 | c) 48 | d) 227 |
| 2) $76 - 5 \cdot 2 \cdot 3$ | a) 198 | b) 66 | c) 426 | d) 46 |
| 3) $\frac{9-2}{4+3}$ | a) 7 | b) 3 | c) 9 | d) 1 |
| 4) $(8+1)(8-1)$ | a) 63 | b) 15 | c) 64 | d) 65 |

Find the area of the shaded region:

5)



- a) 168 square cm. b) 756 square cm. c) 178 square cm. d) 84 square cm.

Find the average:

- 6) 11, 16, 15, 22 a) 64 b) 15 c) 17 d) 16

Solve:

- 7) For five mathematics tests your scores were 81, 86, 81, 76, 71. What was your average score? a) 383 b) 78 c) 79 d) 75

Choose a strategy and solve:

- 8) Your car gets about 20 miles per gallon. You are planning to drive to see your friends who live about 850 miles away. How many gallons of gas will you need to purchase to make the trip to see your friends and to return home?
a) About 20 gallons b) About 41 gallons c) About 85 gallons d) About 48 gallons
- 9) In 2006 your car cost \$13,350. In 2008 your car cost \$17,376. How much did the price increase? a) \$3026 b) \$4026 c) None, it was a decrease d) \$5026
- 10) While shopping for CDs, you note that the average price is about \$8 per CD including tax. You have \$136 in your pocket. About how many CDs can you buy?
a) 21 b) 14 c) 17 d) 19
- 11) In 2007 you weighed 207 pounds. In 2008 you weighed 198 pounds, and in 2009 you weighed 185 pounds. How many pounds did you lose from 2007 to 2009?
a) 22 lbs b) None, you gained weight c) 9 lbs d) 13 lbs

Solve:

12) Write this expression in words:

$$9 + 17 = 26$$

a) The sum of 9 and 17 is 26

b) The difference between 9 and 17 is 26

c) The product of 9 and 17 is 26

d) The quotient of 9 and 17 is 26

Identify a fraction or mixed number that represents the shaded part of the figure:

13)



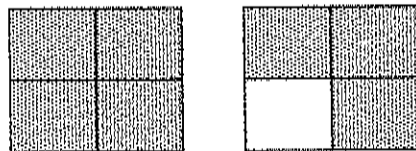
a) 5

b) $\frac{1}{5}$

c) $\frac{1}{6}$

d) $\frac{5}{6}$

14)



a) $\frac{7}{8}$

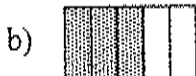
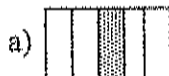
b) $1\frac{3}{4}$

c) 7

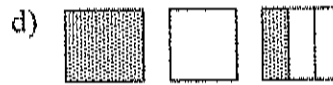
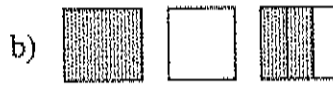
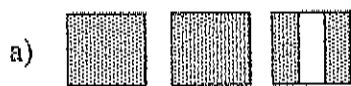
d) $\frac{1}{7}$

Which diagram represents the number:

15) $\frac{3}{5}$



16) $2\frac{1}{3}$



17) $\frac{5}{3}$



Indicate whether the number is a proper fraction, an improper fraction, or a mixed number:

18) $70\frac{11}{17}$

a) Improper fraction

b) Mixed number

c) Proper fraction

Write the mixed number as an improper fraction:

19) $5\frac{4}{7}$

a) $\frac{35}{7}$

b) $\frac{39}{4}$

c) $\frac{35}{4}$

d) $\frac{39}{7}$

Write the improper fraction as a mixed number:

20) $\frac{37}{5}$

a) $7\frac{2}{5}$

b) $7\frac{2}{7}$

c) $8\frac{2}{5}$

d) $6\frac{2}{5}$

Find the value of n:

21) $\frac{5}{16} = \frac{n}{80}$ a) 80 b) 25 c) 5 d) 400

22) $3 = \frac{n}{5}$ a) 15 b) $\frac{5}{3}$ c) $\frac{1}{15}$ d) $\frac{3}{5}$

Simplify:

23) $\frac{12}{20}$ a) $\frac{4}{5}$ b) $\frac{3}{4}$ c) $\frac{12}{20}$ d) $\frac{3}{5}$

24) $\frac{60}{105}$ a) $\frac{4}{15}$ b) $\frac{4}{7}$ c) $\frac{15}{7}$ d) $\frac{60}{105}$

25) $\frac{60}{36}$ a) $\frac{5}{3}$ b) 5 c) $\frac{3}{5}$ d) 15

Between the pair of fractions, insert the appropriate sign: < = >

26) $\frac{1}{2}$ $\frac{3}{8}$ a) < b) > c) =

27) $\frac{4}{16}$ $\frac{4}{13}$ a) = b) > c) <

28) $\frac{5}{7}$ $\frac{15}{21}$ a) < b) > c) =

Solve: Write your answer in simplest form:

29) A baseball team has played 9 games so far this season. The team won 7 games. What fraction of its games has the team won?

a) $\frac{9}{7}$ b) $\frac{7}{9}$ c) $\frac{16}{7}$ d) $\frac{7}{16}$

30) Of a family's \$855 weekly income, \$86 usually goes toward groceries. What fraction of the family's weekly income is usually spent on groceries?

a) $\frac{855}{86}$ b) $\frac{86}{855}$ c) $\frac{43}{385}$ d) $\frac{385}{43}$

31) You have three bolts that are $\frac{3}{8}$ in., $\frac{5}{16}$ in., and $\frac{5}{8}$ in. long. You select the shortest of these to join two plates. Which length is selected?

a) $\frac{5}{8}$ inch

b) $\frac{5}{16}$ inch

c) $\frac{3}{8}$ inch

32) A broker has an order to sell 100 shares of XYZ Company stock if the price rises another $\frac{3}{16}$ of a point. The stock went up $\frac{7}{32}$ points today. Does the broker see the stock?

a) Yes, $\frac{7}{32}$ is greater than $\frac{3}{16}$, so the stock gained enough to sell.

b) No, $\frac{7}{32}$ is less than $\frac{3}{16}$, so the stock didn't gain enough to sell.

Add and simplify:

33) $\frac{3}{8} + \frac{3}{8}$

a) $\frac{2}{3}$

b) $\frac{4}{5}$

c) $\frac{3}{4}$

d) $\frac{2}{4}$

34) $\frac{15}{59} + \frac{9}{59} + \frac{10}{59}$

a) $\frac{34}{177}$

b) $\frac{24}{59}$

c) $\frac{34}{59}$

d) $\frac{61}{59}$

35) $\frac{1}{6} + \frac{2}{7}$

a) $\frac{19}{42}$

b) $\frac{10}{21}$

c) $\frac{3}{13}$

d) $\frac{3}{7}$

36) $\frac{2}{3} + \frac{1}{12}$

a) $\frac{3}{4}$

b) $\frac{7}{12}$

c) $\frac{9}{12}$

d) $\frac{1}{4}$

37) $2\frac{7}{11} + 1\frac{7}{11}$

a) $9\frac{7}{11}$

b) $4\frac{3}{11}$

c) $9\frac{5}{11}$

d) $9\frac{16}{11}$

38) $5\frac{1}{3} + 17\frac{2}{7}$

a) $23\frac{13}{21}$

b) $22\frac{13}{21}$

c) $5\frac{13}{21}$

d) $21\frac{13}{21}$

39) $2\frac{7}{8} + 3\frac{1}{5} + \frac{1}{2}$

a) $7\frac{23}{40}$

b) $5\frac{23}{40}$

c) $6\frac{1}{2}$

d) $6\frac{23}{40}$

Subtract and simplify:

40) $\frac{4}{8} - \frac{3}{8}$

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) $\frac{1}{8}$

d) $\frac{3}{16}$

$$41) \frac{28}{13} - \frac{5}{13} \quad \text{a) } 2\frac{7}{13} \quad \text{b) } \frac{1}{2} \quad \text{c) } \frac{2}{3} \quad \text{d) } 1\frac{10}{13}$$

$$42) \frac{5}{7} - \frac{1}{2} \quad \text{a) } \frac{4}{9} \quad \text{b) } \frac{3}{14} \quad \text{c) } \frac{1}{7} \quad \text{d) } \frac{4}{7}$$

$$43) \frac{7}{9} - \frac{1}{12} \quad \text{a) } \frac{1}{2} \quad \text{b) } \frac{2}{3} \quad \text{c) } \frac{13}{18} \quad \text{d) } \frac{25}{36}$$

$$44) 16\frac{3}{8} - 9\frac{5}{8} \quad \text{a) } 6\frac{2}{4} \quad \text{b) } 6\frac{3}{4} \quad \text{c) } 25\frac{3}{4} \quad \text{d) } 24\frac{3}{4}$$

$$45) 14\frac{2}{7} - \frac{6}{7} \quad \text{a) } 12\frac{3}{7} \quad \text{b) } 13\frac{3}{7} \quad \text{c) } 13\frac{2}{7} \quad \text{d) } 14\frac{3}{7}$$

$$46) 10 - 5\frac{3}{7} \quad \text{a) } 9\frac{4}{7} \quad \text{b) } 5\frac{4}{7} \quad \text{c) } 4\frac{4}{7} \quad \text{d) } 5\frac{3}{7}$$

$$47) 15\frac{2}{7} - \frac{11}{14} \quad \text{a) } 13\frac{1}{2} \quad \text{b) } 14\frac{1}{2} \quad \text{c) } 15\frac{1}{2} \quad \text{d) } 14$$

Solve: Write your answer in simplest form:

48) There were $28\frac{1}{4}$ yards of wire on a spool. After a customer bought $3\frac{5}{8}$ yards of wire from the spool, how many yards were left?

$$\text{a) } 24\frac{5}{8} \text{ yards} \quad \text{b) } 24 \text{ yards} \quad \text{c) } 23\frac{5}{8} \text{ yards} \quad \text{d) } 25\frac{5}{8} \text{ yards}$$

49) Brian was training to run a marathon. During the three-day period before the race he decided that he would train for a total of 11 hours. If he trained for $2\frac{3}{5}$ hours on the first day and $2\frac{9}{10}$ hours on the second day, how many hours would he need to train on the third day?

$$\text{a) } 5\frac{4}{5} \text{ hours} \quad \text{b) } 5\frac{1}{2} \text{ hours} \quad \text{c) } 5\frac{3}{5} \text{ hours} \quad \text{d) } 6\frac{1}{2} \text{ hours}$$

Multiply:

$$50) \frac{3}{8} \cdot \frac{1}{3} \quad \text{a) } \frac{3}{24} \quad \text{b) } \frac{4}{11} \quad \text{c) } \frac{3}{11} \quad \text{d) } \frac{1}{8}$$

- 51) $11 \cdot \frac{5}{8}$ a) $\frac{5}{88}$ b) 88 c) $\frac{55}{8}$ d) $\frac{55}{88}$
- 52) $2\frac{4}{5} \cdot \frac{2}{7}$ a) $\frac{2}{5}$ b) $2\frac{8}{35}$ c) $4\frac{4}{5}$ d) $\frac{4}{5}$
- 53) $4\frac{2}{3} \cdot 9$ a) $13\frac{2}{3}$ b) 42 c) 36 d) 108

Divide:

- 54) $\frac{6}{9} \div \frac{5}{8}$ a) $\frac{16}{15}$ b) $14\frac{2}{5}$ c) $\frac{5}{72}$ d) $\frac{11}{17}$
- 55) $\frac{1}{2} \div 8$ a) $\frac{5}{18}$ b) $\frac{1}{16}$ c) $\frac{1}{2}$ d) None of these
- 56) $17 \div 4\frac{1}{4}$ a) $2\frac{1}{2}$ b) 4 c) 5 d) 3

Solve the problem:

57) Jim has traveled $\frac{5}{6}$ of his total trip. He has traveled 520 miles so far. How many more miles does he have to travel?

- a) 624 miles b) $86\frac{2}{3}$ miles c) None of these d) 104 miles

58) A bag of chips 24 ounces. A serving size is $\frac{3}{4}$ of an ounce. How many servings are in the bag of chips?

- a) 18 servings b) $9\frac{1}{3}$ servings c) 32 servings d) $6\frac{3}{4}$ servings

Write the decimal as a fraction or mixed number in lowest terms:

- 59) 0.14 a) $\frac{1}{196}$ b) $\frac{1}{14}$ c) $\frac{7}{500}$ d) $\frac{7}{50}$
- 60) 13.6 a) $1\frac{9}{25}$ b) $1\frac{3}{5}$ c) $13\frac{3}{5}$ d) $\frac{68}{5}$

Write the number in decimal notation:

- 61) Eight and seventeen hundredths a) 8.0017 b) 8.017 c) 817 d) 8.17

The following sentence involves decimals. Write the decimal in words:

- 62) The weight of a full grown Great Pyrenees dog named Simba is 152.86 pounds.

- a) One thousand fifty-two and eighty-six tenths
- b) One hundred fifty and two hundred eighty-six thousandths
- c) One hundred fifty-two and eighty-six hundredths
- d) One hundred fifty-two and eighty-six thousandths

Write the number in decimal notation:

- 63) A piece of paper is thirty-two thousandths of an inch thick.

- a) 0.032 inches b) 0.00032 inches c) 0.32 inches d) 0.0032 inches

Identify the place value of the underlined digit:

- 64) 0.947 a) Thousandths b) Ones c) Hundredths d) Ten-thousandths

Between each pair of numbers, insert the appropriate sign: $< = >$

- 65) 0.042 0.42 a) $>$ b) $=$ c) $<$

- 66) 8.53 8.503 a) $=$ b) $>$ c) $<$

Rearrange the group of numbers from smallest to largest:

- 67) 2.04, 2.004, 2

- a) 2, 2.04, 2.004 b) 2.04, 2.004, 2 c) 2, 2.004, 2.04 d) 2.004, 2.04, 2

Give an appropriate answer:

- 68) Last summer, your average daily electric bill was for 9.02 units of electricity. This summer, it was for 9.04 units. During which summer was the electrical usage higher?

- a) Last summer b) The usage was the same for both summers
- c) This summer d) Not enough information to determine

Round as indicated:

- 69) 8.628 (nearest hundredth) a) 8.63 b) 8.62 c) 8.64

- 70) 8.73 (nearest tenth) a) 8.8 b) 8.7 c) 8.6

- 71) \$0.07787 (nearest cent) a) \$0.07 b) \$0.00 c) \$0.08 d) \$1.00
 72) 1.942% (nearest tenth) a) 2% b) 1.94% c) 1.9% d) 1.8%
 73) \$21,443.17 (nearest hundred) a) \$21,440 b) \$21,500 c) \$21,400 d) \$21,300

Write the ratio in simplest form:

- 74) 24 to 16 a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{3}{2}$ d) $\frac{3}{4}$

Indicate whether the statement is True or False:

75) $\frac{25}{30} = \frac{3}{5}$

- 76) 5.4 is to 0.6 as 12.6 is to 1.4

Change the percent to a fraction or mixed number. Simplify if necessary:

- 77) 20% a) 2 b) $\frac{1}{10}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$
 78) 300% a) 6 b) 30 c) $\frac{3}{2}$ d) 3
 79) $\frac{7}{10}\%$ a) $\frac{7}{10}$ b) $\frac{7}{1000}$ c) 7 d) $\frac{1}{10}$

Solve the problem:

- 80) What is 30% of 500? a) 150 b) 1.5 c) 1500 d) 15
 81) What is 0.5% of 3200? a) 16 b) 2 c) 160 d) 1600
 82) Compute 150% of 3330 trees:
 a) 49,950 trees b) 4995 trees c) 499,500 trees d) 500 trees
 83) Compute $2\frac{1}{5}\%$ of 83 feet:
 a) 1.83 feet b) 0.02 feet c) 183 feet d) 18.3 feet

Find the percent of increase or decrease:

84) Original value: \$20 New value: \$28

- a) 45% increase b) 45% decrease c) 40% increase d) 40% decrease

85) Original value: \$40 New value: \$12

- a) 70% decrease b) 68% increase c) 80% decrease d) 70% increase

Find the mean of the set of numbers:

86) \$15, \$11, \$6, \$12, \$4, \$4, \$6 (Round to nearest dollar)

- a) \$7 b) \$15 c) \$4 d) \$8

87) 2.7, 6.5, 7.2, 2.7, 4.2 a) 5.825 b) 23.30 c) 4.2 d) 4.66

Use the table to solve the problem:

88)

Moons	Average distance from Geo I (km)	Diameter (km)	Time of Revolution in Earth years
Luna 1	1000	411	0.25
Luna 2	1300	2175	0.77
Luna 3	90,000	314	1.36
Luna 4	129,600	725	43.83
Luna 5	297,000	1136	112.86

What is the time of revolution around Geo I of the moon Luna 4?

- a) 0.77 years b) 43.83 years c) 112.86 years d) 725 years

89)

Lake	Area
Long Lake	183
Big Horn Lake	886
Green Lake	80
Pokagon Lake	890
Thomas Lake	181

How many times as large as the smallest lake is the largest lake?

- a) 0.09 times b) 810 times c) 11.13 times d) 10 times

90) Use the following table to determine the minimum payment on a credit card bill:

Balance	\$0 - \$25	\$25.01 - \$250	\$250.01 - \$1000	\$1000.01 and up
Minimum payment that must be paid	Full balance	\$25	10% of balance	\$100 + 5% of balance greater than \$1000

What is the minimum payment if you have a balance of \$300?

- a) \$300 b) 45 c) \$25 d) \$30

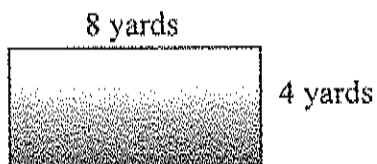
Change the given quantity to the indicated unit:

91) 300 seconds = _____ minutes a) 9 b) 5 c) 12 d) 2

92) 180 inches = _____ feet a) 60 b) 1.25 c) 540 d) 15

Find the perimeter:

93)



- a) 12 yards b) 16 yards c) 8 yards d) 24 yards

Find the square root:

94) $\sqrt{49}$ a) 14 b) 9 c) 24.5 d) 7

95) $\sqrt{144}$ a) 144 b) 72 c) 12 d) 24

Answer key for Part 1 – General Mathematics section:

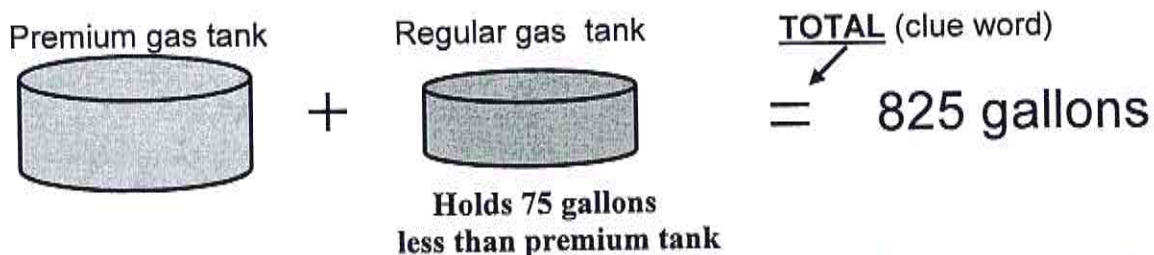
- | | | |
|-------|-------|-----------|
| 1) D | 34) C | 67) C |
| 2) D | 35) A | 68) C |
| 3) D | 36) A | 69) A |
| 4) A | 37) B | 70) B |
| 5) A | 38) B | 71) C |
| 6) D | 39) D | 72) C |
| 7) C | 40) C | 73) C |
| 8) C | 41) D | 74) C |
| 9) B | 42) B | 75) FALSE |
| 10) C | 43) D | 76) TRUE |
| 11) A | 44) B | 77) D |
| 12) A | 45) B | 78) D |
| 13) D | 46) C | 79) B |
| 14) B | 47) B | 80) A |
| 15) B | 48) A | 81) A |
| 16) C | 49) B | 82) B |
| 17) C | 50) D | 83) A |
| 18) B | 51) C | 84) C |
| 19) D | 52) D | 85) A |
| 20) A | 53) B | 86) D |
| 21) B | 54) A | 87) D |
| 22) A | 55) B | 88) B |
| 23) D | 56) B | 89) C |
| 24) B | 57) D | 90) D |
| 25) A | 58) C | 91) B |
| 26) B | 59) D | 92) D |
| 27) C | 60) C | 93) D |
| 28) C | 61) D | 94) D |
| 29) B | 62) C | 95) C |
| 30) B | 63) A | |
| 31) B | 64) A | |
| 32) A | 65) C | |
| 33) C | 66) B | |

BASIC WORD PROBLEMS

Following these steps will help you get through a word problem:

1. Draw a picture that matches your information.
2. Put all information from the problem on your picture. (In English - Not Algebra)
If you can tell what the problem says by looking at your picture, then you have done an excellent job.
OPTION: Make a chart for all your information. (Great organizational tool.)
3. To know that you really understand the problem, try putting in a **reasonable guess** of the answer and then figure out if it is correct or incorrect. You could continue to guess at the answer until you get it correct, but eventually guessing will take up too much of your time.
4. Using **algebra** will allow us to find the correct answer without all the guessing. This means that we will use a **variable to represent the correct answer** in place of the value we were guessing. We will write an equation very similar to the work as when we were guessing. Equation clue words: TOTAL, SUM, or similar words.
5. Solve the algebraic equation. Check your answer.
6. Answer the original question by writing in sentence format.

Example: At a service station, the underground tank storing regular gas holds 75 gallons less than the tank storing premium gas. If the total storage capacity of the two tanks is 825 gallons, how much does the premium gas tank hold?



I am going to **guess** that the premium tank holds **500 gallons**. To check our guess we can calculate that the regular gas tank holds 425 gallons (75 less than the premium tank of 500 gallons). If our guess had been correct the total for the two tanks would be 825. But because 500 and 425 totals to the incorrect value of 925, we need to guess again!

Using **algebra** we can assign a variable (the correct answer) to the amount in the premium tank. The correct amount in the premium tank is **P**. Therefore the amount in the regular tank (which is 75 gallons less) is **P - 75**. Now since these answers are correct the total will be 825 gallons.

Reasoning: Amount in premium tank + Amount in regular tank = 825 gallons
 Equation: $P + P - 75 = 825$
 Solve: $2P - 75 = 825$ (Combine like terms)
 $2P = 900$ (Add 75 to both sides of equation)
 $P = 450$ (Divide both sides by 2)

Answer: The premium tank holds 450 gallons of gas.

WHAT THE WORDS MEAN

Addition:	Sum	the sum of x and 5	$x + 5$
Addition:	Plus	12 plus y	$12 + y$
Addition:	Increased by	h increased by 8	$h + 8$
Addition:	Exceeds	exceeds m by 6	$m + 6$
Addition:	Added to	7 added to m	$m + 7$
Addition:	More than	h more than 4	$4 + h$
Addition:	Greater than	3 greater than y	$y + 3$
Subtraction:	Difference	the difference of k and 23	$k - 23$
Subtraction:	Minus	13 minus x	$13 - x$
Subtraction:	Decreased by	h decreased by 8	$h - 8$
Subtraction:	Reduced by	13 reduced by r	$13 - r$
Subtraction:	Less	24 less m	$24 - m$
Subtraction:	Less than	15 less than c	$c - 15$
Subtraction:	Subtracted from	8 subtracted from r	$r - 8$
Multiplication:	Product	the product of 15 and x	$15x$
Multiplication:	Times	5 times m	$5m$
Multiplication:	Twice	twice w	$2w$
Multiplication:	Of	half of t	$\frac{1}{2}t$
Division:	Per	miles per gallon	miles/gallon
Division:	Quotient	the quotient of r and 7	$r / 7$
Division:	Divided by	4 divided by y	$4 / y$
Division:	Ratio	the ratio of z to 13	$z / 13$
Division:	Split into	5 split into n equal parts	$5 / n$
Exponent:	Square	the square of m	m^2
Exponent:	Cube	the cube of r	r^3
Equals:	Is	The sum of 4 and r is 9	$4 + r = 9$
Equals:	Result is	If you increase m by 7, the <u>result is</u> twice x	$m + 7 = 2x$
Inequality	Is not equal to	y is not equal to twice x	$y \neq 2x$
Inequality	Is less than	6 is less than r	$6 < r$
Inequality	Is greater than	m is greater than y	$m > y$
Inequality		r is less than or equal to z	$r \leq z$
Inequality		p is greater than or equal to w	$p \geq w$

Multiplying & Dividing Fractions and Mixed Numbers

Steps:

1. Rewrite all mixed numbers as improper fractions.
2. If it is a division problem, find the reciprocal of the fraction following the division sign and then multiply.
3. Multiply the numerators.
4. Multiply the denominators.
5. Reduce the fraction, if possible.
6. If the fraction is improper, rewrite as a mixed number.

Multiplication

$$5\frac{3}{5} \cdot 3\frac{1}{4}$$

Example

$$\frac{28}{5} \cdot \frac{13}{4}$$

Step #1

$$\frac{28}{5} \cdot \frac{13}{4}$$

Step #2

$$\frac{364}{20}$$

Step #3 & #4

$$\frac{91}{5}$$

Step #5

$$18\frac{1}{5}$$

Step #6

Division

$$2\frac{5}{8} \div \frac{3}{4}$$

$$\frac{21}{8} \div \frac{3}{4}$$

$$\frac{21}{8} \cdot \frac{4}{3}$$

$$\frac{84}{24}$$

$$\frac{7}{2}$$

$$3\frac{1}{2}$$

Easier Option:

Reduce any common factors before multiplying.

$$\frac{28}{5} \cdot \frac{13}{4}$$

Step #2

$$\frac{21}{8} \cdot \frac{4}{3}$$

$$\frac{7}{5} \cdot \frac{13}{1}$$

Step #5

$$\frac{7}{2} \cdot \frac{1}{1}$$

$$\frac{91}{5} = 18\frac{1}{5}$$

Step #6

$$\frac{7}{2} = 3\frac{1}{2}$$

Adding and Subtracting Like Fractions

We can ONLY add and subtract the same thing. In the world of fractions the denominators must be the same to have LIKE fractions.

Steps:

1. Add or subtract the numerators (How many units you are adding or subtracting).
2. Write down the denominator (What you are adding or subtracting).
3. Reduce your fraction to lowest terms.
4. If it is improper, write it as a mixed number.

Examples with steps #1 and #2 only:

$$\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$$

$$\frac{7}{11} - \frac{4}{11} = \frac{3}{11}$$

$$\frac{1}{7} + \frac{3}{7} + \frac{2}{7} = \frac{6}{7}$$

$$\frac{12}{13} - \frac{2}{13} = \frac{10}{13}$$

Examples that include step #3 and/or step #4:

$$\frac{9}{10} - \frac{1}{10} = \frac{8}{10} = \frac{4 \cdot 2}{5 \cdot 2} = \frac{4}{5}$$

To reduce a fraction we need to find a number that will divide evenly into the numerator and denominator. In the example above 2 goes into 8 and 10 evenly. Because the $\frac{2}{2}$ has a value of 1 and multiplying by one does not change the value of the expression, we can drop it from the problem.

$$\frac{13}{9} - \frac{1}{9} = \frac{12}{9} = \frac{4 \cdot 3}{3 \cdot 3} = \frac{4}{3} = 1\frac{1}{3}$$

In this example we have to reduce as in the previous example. And then write the final answer as a mixed number because the numerator is greater than the denominator.

Adding and Subtracting Unlike Fractions

Steps:

1. Find the common denominator (the number all denominators go into evenly).
2. Using appropriate identities change all fractions to the common denominator.
3. Add or subtract the numerators (How many units you are combining).
4. Write down the denominator (What you are adding or subtracting).
5. Reduce your fraction to lowest terms.
6. If it is improper, write it as a mixed number.

$$\frac{2}{3} \pm \frac{1}{4}$$

LCD is 12: The smallest number that 3 and 4 both go into evenly.

$$\frac{2}{3} \cdot \frac{4}{4} \pm \frac{1}{4} \cdot \frac{3}{3}$$

Find the appropriate identities that will give a denominator of 12.

$$\frac{8}{12} \pm \frac{3}{12}$$

Multiply by the identities to create **LIKE** denominators.

$$\frac{11}{12}$$

Add numerators, if problem is addition.

$$\frac{5}{12}$$

Subtract numerators, if problem is subtraction.

Subtracting Mixed Numbers

Steps:

1. Make both fractions into like denominators.
2. Subtract the numerators.
If the value is negative, you must borrow one (1) from the whole number.
The borrowed one (1) must be changed to an identity fraction with the same denominator as the fraction and then added together.
Now you can subtract the numerators and get a positive value.
3. Subtract the whole numbers.
4. Reduce the fraction, if possible.

$$45\frac{1}{4} - 12\frac{3}{8}$$

Example

$$\begin{array}{r} 45\frac{1}{4} = 45\frac{2}{8} \\ -12\frac{3}{8} = -12\frac{3}{8} \\ \hline \end{array}$$

Rewrite vertically and change into like denominators.

Since subtraction at this point would create a negative fraction, we need to borrow one from the whole number (45) to keep our values positive.

$$\begin{array}{r} 1 = \frac{8}{8} \\ \text{Borrow 1} \swarrow \\ 45\frac{2}{8} = 44\frac{2}{8} + \frac{8}{8} = 44\frac{10}{8} \\ -12\frac{3}{8} \\ \hline 32\frac{7}{8} \end{array}$$

$\left[\frac{8}{8}\right]$ was chosen because we can only add like fractions.

Borrow 1 from 45 and write as an identity fraction with the same denominator. Add numerators.

Subtract the fractions and whole numbers.
This fraction cannot be reduced.

Working With Decimals

Assumption: When we do not see a decimal in a number it is assumed that it is after the last digit of the number.

Place value:

2 9 4 8 . 5 7 1 6

thousands hundreds tens ones tenths hundredths thousandths ten-thousandths

values greater than one . fractional values

Rounding off: 265.273 Round off to nearest tenths.
The correct answer will be 265.3 because the digit after the tenths column is 5 or higher.

5089.8924 Round off to nearest thousandths.
The correct answer will be 5089.892 because the digit after the thousandths column is below 5.

Adding / Subtracting: *Line up the decimal points to make like columns.*
When subtracting you should first put zeros in all empty columns.

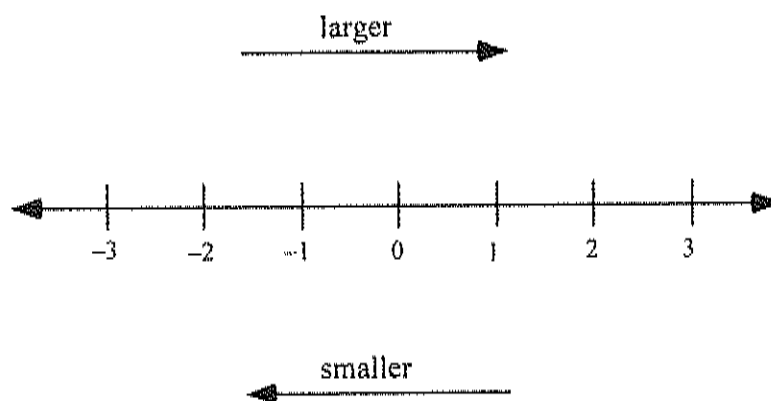
Multiplying:

- Step 1: Multiply the numbers as though there are no decimal points.
- Step 2: Total up the number of digits after both decimal points.
- Step 3: Put the decimal in the answer so that the number of digits after the decimal will be the same as your answer to step #2.

Dividing: The divisor (number you are dividing by) must be a natural number.

- Step 1: If the divisor is not a natural number, then move the decimal all the way to the right.
- Step 3: If you move the decimal in the divisor, you must also move the decimal in the dividend (number you are dividing into) the same amount of places. Zeros may be added, if needed.
- Step 3: Divide normally.
- Step 4: Zeros can be added to the dividend to get appropriate answer.

Putting Numbers in Order by Relative Size



- Comparing fractions: Must have common denominators.

Put in proper inequality: $\frac{3}{4}$ $\frac{5}{6}$

Rewrite with common denominator: $\frac{9}{12}$ $\frac{10}{12}$

Because 9 is smaller than 10: $\frac{9}{12} < \frac{10}{12}$

- Comparing decimals: Must look at common columns.

Put in proper inequality: 34.654 34.628

Look at each common column individually starting from the left.

Tens column: Both are equal (3).

Ones column: Both are equal (4).

Tenths column: Both are equal (6).

Hundredths column: 5 hundredths is greater than 2 hundredths.

Thousandths column: Since we have already found a larger column that determines which is greater, this is unimportant!

Therefore: 34.654 > 34.628

- To compare more than 2 fractions or decimals you can use the same procedure as above to find the largest or smallest value.

Percentage Information

Percentage means: Out of 100 or .01 or $\frac{1}{100}$

$$\text{Number \% of Number} = \text{Number}$$

25 % of 80 is what?	30 % of what is 21?	What % of 50 is 10?
$25(.01)(80) = \text{number}$	$30(.01)(\text{number}) = 21$	$\text{number}(.01)(50) = 10$
$(25)(.01)(80) = \text{number}$	$(30)(.01)(\text{number}) = 21$	$(\text{number})(.01)(50) = 10$
20 = number	$(.30)(\text{number}) = 21$	$(\text{number})(.50) = 10$
	number = 70	number = 20
	(Divided both sides by .3)	(Divided both sides by .5)

Therefore:

25% of 80 is 20

30% of 70 is 21

20% of 50 is 10

Percentage is less than one (1):

0.25 % of 75 is what number?	$\frac{3}{4}$ % of 20 is what number?
$0.25(.01)(75) = \text{number}$	$(\frac{3}{4})(\frac{1}{100})(20) = \text{number}$
0.1875 = number	$\frac{3}{20} = \text{number}$

Percentage is between 1 and 99:

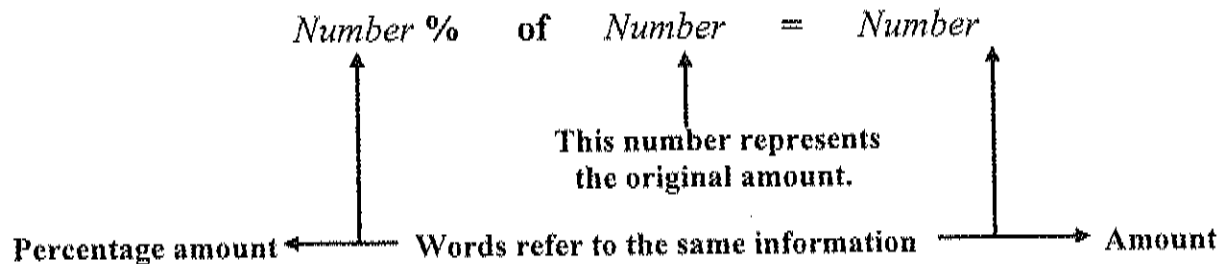
45.8% of 39 is what number?	$4\frac{2}{5}$ % of 50 is what number?
$(45.8)(.01)(39) = \text{number}$	$\frac{22}{5}$ % of 50 is what number?
17.862 = number	$(\frac{22}{5})(\frac{1}{100})(50) = \text{number}$
	$2\frac{1}{5} = \text{number}$

Percentage is greater than 100:

230 % of 65 is what number?	580.65 % of 207 is what number?
$(230)(.01)(65) = \text{number}$	$(580.65)(.01)(207) = \text{number}$
149.5 = number	1201.9455 = number

Percentage Applications

Percentage means: Out of 100 or .01 or $\frac{1}{100}$



Example 1: Finding the amount

John owns a citrus grove with 200 trees. If 25% of his trees are grapefruit, how many grapefruit trees does John have?

25% of 200 citrus trees are grapefruit trees.

$$25\% \text{ of } 200 = g$$

$$(25) (.01) (200) = g$$

$$50 = g$$

Therefore John must have 50 grapefruit trees in this grove.

Reminder: The word "of" means that we should be multiplying.

Example 2: Finding the percentage amount

Mary has a plant nursery. If 200 of the 1000 plants she grows are trees, what percent of her nursery are trees?

What percent of 1000 plants are 200 trees?

$$n \% \text{ of } 1000 = 200$$

$$n (.01) (1000) = 200$$

$$n (10) = 200$$

$$n = 20 \quad (\text{We divided both sides by } 10)$$

Therefore 20% of the nursery was trees.

Example 3: Finding the original amount

In the last election 30% of the people eligible to vote voted. If 1500 people voted, how many people were eligible to vote?

30% of the eligible voters are the 1500 people who voted.

$$30\% \text{ of } v = 1500$$

$$(30) (.01) v = 1500$$

$$(0.3) v = 1500$$

$$v = 5000 \quad (\text{We divided both sides by } 0.3)$$

Therefore there were 5000 people eligible to vote in the election.

Consumer Applications: Simple Interest

I represents the simple Interest you will receive.

P represents the Principal which is how much you have in your account.

R represents the Rate which is how much you will get at the end of a year for every \$100 you have in your account.

T represents the amount of Time (in years) that you leave your money in the account.

$$I = PRT$$

Example 1:

What is the interest earned on \$3000 invested at 6% for 3 years?

Principal = \$3000 (This is the amount in your account.)

$R = 6\%$ or .06 per year

$T = 3$ years

$$I = P \cdot R \cdot T$$

$$I = (3000)(.06)(3)$$

$$I = 540$$

The interest earned is \$540 over a 3-year period.

Example 2:

What is the interest earned on \$200 invested at 9% for 8 months?

Principal = \$200 (This is the amount in your account.)

$R = 9\%$ or .09 per year.

$T = \frac{8}{12}$ of a year. (Remember time is always per year.)

$$I = P \cdot R \cdot T$$

$$I = (200)(.09)(\frac{8}{12})$$

$$I = 12$$

The interest earned is \$12 for a period of 8 MONTHS.

Part 2

Developmental Math I

MAT0018

Previously called Prealgebra MAT0012

NOTE: The material in this section will not specifically be on the PERT test. But if you do not know this material then the possibility of getting into a higher course is unlikely.

Test for Developmental Math I

- 1a. Place the following numbers in order from least to greatest: $-5, 2, -8, 0, 5$
- a. $-5, -8, 0, 2, 5$
 - b. $5, 2, 0, -5, -8$
 - c. $-8, -5, 0, 2, 5$
 - d. $0, -8, -5, 5, 2$
- 1b. Place the following numbers in order from least to greatest: $-1.5, -1.05, -1.052$
- a. $-1.5, -1.052, -1.05$
 - b. $-1.05, -1.052, -1.5$
 - c. $-1.05, -1.5, -1.052$
 - d. $-1.5, -1.05, -1.052$
- 1c. Place the following numbers in order from least to greatest: $\frac{3}{5}, \frac{2}{3}, \frac{1}{2}, \frac{3}{4}$
- a. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{3}{5}$
 - b. $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}$
 - c. $\frac{2}{3}, \frac{3}{5}, \frac{3}{4}, \frac{1}{2}$
 - d. $\frac{3}{5}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$
- 2a. Simplify: $5 + (-3) - (-6)$
- a. -4
 - b. 2
 - c. 8
 - d. 23
- 2b. Simplify: $\frac{8 + 3(-4)}{-2}$
- a. 2
 - b. -2
 - c. 11
 - d. 14

2c. Simplify: $-2 \cdot 3 - 15 \div (-5)4 - (-7)$

- a. -11
- b. 13
- c. 19
- d. -99

3a. Simplify: $-6x^2 - 3x + 7x^2 - 5x$

- a. $-7x^2$
- b. $x^2 - 2x$
- c. $x^2 - 8x$
- d. $13x^2 + 2x$

3b. Simplify: $5x^3 + 7xy - 2x^2 - xy + x^3$

- a. $4x^3 + 2x^2 + 7$
- b. $6x^3 - 2x^2 + 7$
- c. $5x^6 - 2x^2 + 8xy$
- d. $6x^3 - 2x^2 + 6xy$

4a. Simplify: $-3(x - 5)$

- a. $-3x - 5$
- b. $-3x - 15$
- c. $-3x + 15$
- d. $-3x - 8$

4b. Simplify: $(x + 6)(x + 5)$

- a. $x^2 + 11x + 30$
- b. $x^2 + 30x + 30$
- c. $x^2 + 11$
- d. $x^2 + 30$

4c. Simplify: $(2x - 5)^2$

- a. $4x^2 + 25$
- b. $2x^2 + 25$
- c. $4x^2 - 20x + 25$
- d. $4x^2 - 25$

5a. Evaluate the algebraic expression when $a = -5$, $b = 2$, and $c = -7$:

$$a^2 + b - c$$

- a. -1
- b. 5
- c. 20
- d. 34

6a. Simplify: $2y(3 - x) + 7(x - 2y)$

- a. $12y - 2xy + x$
- b. $xy + 7x - 8y$
- c. $7x - 2xy - 8y$
- d. $4y + 6x$

6b. Simplify: $(3m^2 + 5m - 7) - (4m^2 - 2m + 9)$

- a. $-m^2 + 7m - 16$
- b. $-m^2 + 3m + 2$
- c. $7m^2 - 3m + 16$
- d. $7m^2 + 7m - 16$

7a. Solve: $8(x - 2) = 5(x + 4)$

- a. $x = 2$
- b. $x = 12$
- c. $x = \frac{4}{3}$
- d. $x = \frac{4}{11}$

7b. Solve: $-5(3x + 4) = 2(-x + 3)$

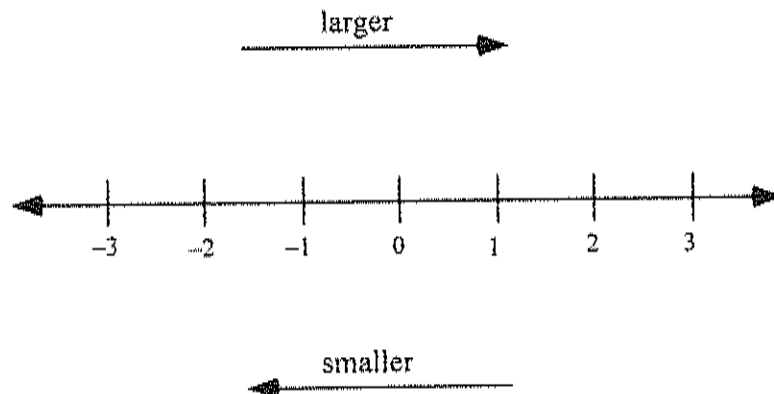
- a. $x = -2$
- b. $x = \frac{14}{17}$
- c. $x = \frac{1}{13}$
- d. $x = 2$

Developmental Math I Test Answers:

- 1a. c
- 1b. a
- 1c. b
- 2a. c
- 2b. a
- 2c. b
- 3a. c
- 3b. d
- 4a. c
- 4b. a
- 4c. c
- 5a. d
- 6a. c
- 6b. a
- 7a. b
- 7b. a

DM-I Question #1:

Putting Numbers in Order by Relative Size



- Comparing fractions: Must have common denominators.

Put in proper inequality: $\frac{3}{4}$ $\frac{5}{6}$

Rewrite with common denominator: $\frac{9}{12}$ $\frac{10}{12}$

Because 9 is smaller than 10: $\frac{9}{12} < \frac{10}{12}$

- Comparing decimals: Must look at common columns.

Put in proper inequality: 34.654 34.628

Look at each common column individually starting from the left.

Tens column: Both are equal (3).

Ones column: Both are equal (4).

Tenths column: Both are equal (6).

Hundredths column: 5 hundredths is greater than 2 hundredths.

Thousandths column: Since we have already found a larger column that determines which is greater, this is unimportant!

Therefore: 34.654 > 34.628

- To compare more than 2 fractions or decimals you can use the same procedure as above to find the largest or smallest value.

DM-I Question #2: Operations with Integers

- Integers are positive and negative whole numbers including zero.
- Set of integers: $\{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$
- Integers do **NOT** include fractions or decimals.

Addition:

- Positive integers: How much money you have!
- Negative integers: How much money you owe!
- Addition is the process of combining what you have and owe.
- | <u>Example</u> | <u>Read as follows</u> | <u>Read answer</u> | <u>Answer</u> |
|----------------|---------------------------|--------------------|---------------|
| $-6 + 8$ | You owe \$6 and have \$8 | Have \$2 | 2 |
| $3 + (-7)$ | You have \$3 and owe \$7 | Owe \$4 | -4 |
| $-5 + (-2)$ | You owe \$5 and owe \$2 | Owe \$7 | -7 |
| $6 + 4$ | You have \$6 and have \$4 | Have \$10 | 10 |

Subtraction:

- Are you having trouble doing a subtraction problem? If so, then do the opposite of what follows the subtraction sign and work the problem as an addition problem.
- | <u>Example</u> | <u>Written as addition</u> | <u>Answer</u> |
|----------------|----------------------------|---------------|
| $4 - 9$ | $4 + (-9)$ | -5 |
| $7 - (-2)$ | $7 + 2$ | 9 |
| $-5 - 8$ | $-5 + (-8)$ | -13 |
| $-3 - (-7)$ | $-3 + 7$ | 4 |

Multiplication and Division:

- LIKE signs give a positive answer.
- UNLIKE signs give a negative answer.
- | <u>Example</u> | <u>Answer</u> | | <u>Example</u> | <u>Answer</u> |
|----------------|---------------|--------------|----------------|---------------|
| $4 \cdot 5$ | 20 | Like signs | $10 \div 5$ | 2 |
| $-3 \cdot -8$ | 24 | Like signs | $-6 \div (-2)$ | 3 |
| $7 \cdot -6$ | -42 | Unlike signs | $8 \div (-4)$ | -2 |
| $-4 \cdot 2$ | -8 | Unlike signs | $-9 \div 3$ | -3 |

DM-I Question #3:

Adding and Subtracting Polynomials Terms

When adding or subtracting polynomial terms, the terms must be like terms. Like terms are defined as algebraic terms which have the same variable(s) with the same exponents.

Simplify: $-6x^2 - 3x + 7x^2 - 5x$
 $-6x^2 + 7x^2 - 3x - 5x$

Example

Use commutative property to rearrange.

$-6x^2$ and $7x^2$ both have " x^2 " as the like term

$-3x$ and $-5x$ both have " x " as the like term

$$x^2 - 8x$$

Total up the x^2 terms: $-6x^2 + 7x^2 = 1x^2$

Total up the x terms: $-3x - 5x = -8x$

Which are like terms?

Remember that like terms must have the same variable(s) with the same exponents.

The following are like terms because of this fact.

$4x$ and $-16x$ are like terms.
 $6xy^2$ and $5xy^2$ are like terms.
 $-3x^3$ and $14x^3$ are like terms.

The following are not like terms:

$4x$ and $8y$ Because they have different variables.

$6x^2y$ and $8x^2y^2$ Because the exponents are not the same on the " y " variable.

$8x^3$ and $7x^2$ Because the exponents are not the same on the " x " variable.

Simplify:

1. $5x - 3x^2 + 7x - 3x^3$

2. $6x + 7x^2 - 3x^2 + 2x$

3. $15x - 6x - 5xy^2 + 8x^2y$

4. $3x + 5x^2 + 6x^2 - x$

5. $4x^2 - 3x + 9x - 8x^2$

Answers:

$-3x^3 - 3x^2 + 12x$

$4x^2 + 8x$

$8x^2y - 5xy^2 + 9x$

$11x^2 + 2x$

$-4x^2 + 6x$

DM-I Question #4:

Multiplying Polynomials

Monomial (one term) times Binomial (two terms):

Simplify: $5(2x + 7)$ Example
 $5 \cdot 2x + 5 \cdot 7$ Distributive property
 $10x + 35$ Multiplication of factors

Simplify: $(5x - 3)(-4y)$ Example
 $(-4y)(5x) - (-4y)(3)$ Distributive property
 $-20xy + 12y$ Multiplication of factors

Binomial (2 terms) times Binomial (2 terms):

Simplify: $(2x + 7)(5x + 3)$

1. Multiply First terms
2. Multiply Outside terms
3. Multiply Inside terms
4. Multiply Last terms

$\frac{2x \cdot 5x}{\text{First}} + \frac{2x \cdot 3}{\text{Outside}} + \frac{7 \cdot 5x}{\text{Inside}} + \frac{7 \cdot 3}{\text{Last}}$ Distributive property
Spells "FOIL"

Did you notice that the first letter of each word F-O-I-L spells FOIL???
This is a trick to help you remember the steps to multiplying 2 binomials!

$10x^2 + 6x + 35x + 21$ Multiplication of factors
 $10x^2 + 41x + 21$ Addition of like terms

Binomial (2 terms) times Binomial (2 terms) continued:

These problems use the same method as on the previous page.
But they **LOOK** different because there is no middle term!

Simplify: $(5x + 2)(5x - 2)$

$$5x \cdot 5x - 5x \cdot 2 + 5x \cdot 2 - 2 \cdot 2$$

$$25x^2 - 10x + 10x - 4$$

$$25x^2 - 4$$

Example

Distributive property

Multiplication of factors

Combine like terms

Note: This answer is called: **Difference of squares**
because it is subtraction and both terms are perfect squares.

Simplify: $(x + 4)^2$
 $x^2 + 8x + 16$

MEANS $(x + 4)(x + 4)$

Distribute:

1. $3(2x + 5)$

2. $8(3x - 2)$

3. $-4(5x + 3)$

4. $-5(2x - 5)$

5. $(3x - 2)(-6)$

6. $3x(5x + 2)$

7. $-2x(7x - 4)$

8. $5y(2x + 3m + 6)$

9. $(-x + 5r^2 - 7)(-2)$

10. $4x + 5(2y + 3)$

The $4x$ is a separate term and not part of the distributive property!

11. $(x + 5)(x + 3)$

12. $(3x + 5)(-2x + 1)$

13. $(2x + 3)(4m + 5)$

14. $(3x - 2)(2x - 5)$

15. $(5x^2 + 8)(2x - 3)$

16. $(4x - 3)(4x + 3)$

17. $(5y + 4)(5y - 4)$

18. $(2x - 5)^2$

19. $(7m + 3)^2$

Answers:

$$6x + 15$$

$$24x - 16$$

$$-20x - 12$$

$$-10x + 25$$

$$-18x + 12$$

$$15x^2 + 6x$$

$$-14x^2 + 8x$$

$$10xy + 15my + 30y$$

$$2x - 10r^2 + 14$$

$$4x + 10y + 15$$

$$x^2 + 8x + 15$$

$$-6x^2 - 7x + 5$$

$$8mx + 10x + 12m + 15$$

$$6x^2 - 19x + 10$$

$$10x^3 - 15x^2 + 16x - 24$$

$$16x^2 - 9$$

$$25y^2 - 16$$

$$4x^2 - 20x + 25$$

$$49m^2 + 42m + 9$$

DM-I Question #5:**Evaluating an Expression**

Evaluating an expression is done by replacing a variable with a given value and then simplifying using order of operations.

Evaluate:	$3x + 7$	when $x = 5$
	$3(5) + 7$	Substitute 5 for the variable x
	$15 + 7$	Simplify using order of operations
	22	Simplify using order of operations

Evaluate:	$7x^2 + yz$	when $x = 2$, $y = 7$, and $z = 9$
	$7(2)^2 + (7)(9)$	Substitute 2 for x , 7 for y , and 9 for z
	$7(4) + (7)(9)$	Simplify using order of operations
	$28 + 63$	Simplify using order of operations
	91	Simplify using order of operations

Evaluate:	$2x - 3y$	when $x = 6$ and $y = -4$
	$2(6) - 3(-4)$	Substitute 6 for x and -4 for y
	$12 + 12$	Simplify using order of operations
	24	Simplify using order of operations

Evaluate:	$-3x - 5xy - z^2$	when $x = -2$, $y = 7$, and $z = -9$
	$-3(-2) - 5(-2)(7) - (-9)^2$	Substitute -2 for x , 7 for y , -9 for z
	$-3(-2) - 5(-2)(7) - 81$	Simplify using order of operations
	$6 + 70 - 81$	Simplify using order of operations
	-5	Simplify using order of operations

<u>Evaluate</u>		<u>Answers</u>
1.	$5x - y$ when $x = 3$ and $y = 6$	9
2.	$5g^3p$ when $g = 4$ and $p = 7$	2240
3.	$-2y - z$ when $y = 9$ and $z = -2$	-16
4.	$abc - m^2$ when $a = -4$, $b = 3$, $c = -6$, $m = -5$	47
5.	$-r^2 + 3w^2$ when $r = 8$ and $w = -6$	44

DM-I Question #6:**Simplifying an Expression**

An expression is one or more terms that are added or subtracted.

Examples: $3x$, $2x + 5$, $-6x - 5$, $4x^2 + 3m - 9r + 5$

SIMPLIFY an expression using $GE(\overline{MD})(\overline{AS})$:

1. Work inside all Groupings.

(*Parentheses*), [*Brackets*], $\frac{\text{Fraction}}{\text{Bar}}$, $\sqrt{\text{Radical}}$, $|\text{Absolute value}|$

2. Work with Exponents.

3. Do **M**ultiplication and **D**ivision combinations from Left to Right.

4. Do Addition and Subtraction combinations from Left to Right.

Simplify: $3x - 5x + 2x \cdot 4$

$$3x - 5x + 8x$$

$$6x$$

Multiply

Add / Subtract like terms left to right

Simplify: $5(2x + 7 - 3x) + (4x)^2$

$$5(-1x + 7) + (4x)^2$$

$$5(-1x + 7) + 16x^2$$

$$-5x + 35 + 16x^2$$

Work inside parentheses

Work with exponents

Distributive property (multiplying)

Note: Instructors expect terms with exponents to be written in descending order like this: $16x^2 - 5x + 35$

Simplify: $(5r^2 + 3r - 2) - (2r^2 - 7r + 8)$

$$5r^2 + 3r - 2 - 2r^2 + 7r - 8$$

Distributive property

Note: Subtracting will change the sign of each term being subtracted.

$$3r^2 + 10r - 10$$

Add / Subtract like terms.

<u>Simplify</u>	<u>Answer</u>
1. $3x - 7x + 5 - 8 + x$	$-3x - 3$
2. $(5x + 3) - (3x - 8)$	$2x + 11$
3. $4(2x - 7) + 3x(5x - 2)$	$15x^2 + 2x - 28$
4. $5h + 3(2h - 7) - 2(6h) + 10$	$-h - 11$
5. $-4(3w - 5) - (2w^2 - 8w + 9)$	$-2w^2 - 4w + 11$
6. $(5r + 6 - 11)2 + 8(-3r + r + 2)$	$-6r + 6$

DM-I Question #7:**Solving an Equation**

Equation (2 expressions that are equal to each other):

$$3x + 8 = x - 2$$

Left expression = Right expression

SOLVE: Means to isolate a specific variable in an equation on either side by doing the same thing to both the simplified expression on the left side and the right side.

1. Simplify: $GE(\overline{MD})(\overline{AS})$ the expressions on either side of the equation.
2. Collect all variables into ONE TERM (add or subtract to both sides).
3. Isolate the variable term (add or subtract constant term to both sides).
4. Isolate the variable itself (multiply or divide coefficient to both sides).

Solve: $2(3x + 5) = 46$

$6x + 10 = 46$	Simplify the left side
$6x + 10 - 10 = 46 - 10$	Subtract constant term from both sides
$6x = 36$	Simplify to isolate variable term
$\frac{6x}{6} = \frac{36}{6}$	Divide by coefficient on both sides
$x = 6$	Simplify to isolate variable

Solve: $4(2x - 3) = 5(3x + 2)$

$8x - 12 = 15x + 10$	Simplify both sides
$8x - 12 - 8x = 15x + 10 - 8x$	Subtract $8x$ term from both sides
$-12 = 7x + 10$	Simplify variables into one term
$-12 - 10 = 7x + 10 - 10$	Subtract constant from both sides
$-22 = 7x$	Simplify to isolate variable term
$\frac{-22}{7} = \frac{7x}{7}$	Divide by coefficient on both sides
$\frac{-22}{7} = x$	Simplify to isolate variable

This page shows shortcuts that MAY be used to clear decimals or fractions from an equation before solving!

Solving an equation involving a decimal:

Solve: $3.5x - 1.7 = 13.2$

Note: Since the most digits after any decimal point is one, if we multiply both sides by 10 we can remove the decimals.

$$10(3.5x - 1.7) = (13.2)10 \quad \text{Multiply both sides by 10}$$

$$35x - 17 = 132 \quad \text{Distributive property}$$

Solve the equation as done with integers.

Solve: $2.4x + 45.57 = 9$

Note: Since the most digits after any decimal point is two, if we multiply both sides by 100 we can remove the decimals.

$$100(2.4x + 45.57) = (9)100 \quad \text{Multiply both sides by 100}$$

$$240x + 4557 = 900 \quad \text{Distributive property}$$

Solve the equation as done with integers.

Solving an equation involving a fraction:

Solve: $\frac{2}{3}x + \frac{5}{6} = \frac{3}{4}$

Note: Since the LCD for all fractions is 12, if we multiply both sides by 12 we can remove the denominators by reducing.

$$12\left(\frac{2}{3}x + \frac{5}{6}\right) = 12\left(\frac{3}{4}\right) \quad \text{Multiply both sides by 12 (LCD)}$$

$$\frac{12}{1} \cdot \frac{2}{3}x + \frac{12}{1} \cdot \frac{5}{6} = \frac{12}{1} \cdot \frac{3}{4} \quad \text{Distributive property}$$

$$\frac{4}{1} \cdot \frac{2}{1}x + \frac{2}{1} \cdot \frac{5}{1} = \frac{3}{1} \cdot \frac{3}{1} \quad \text{Reduce fractions}$$

$$8x + 10 = 9 \quad \text{No denominator format}$$

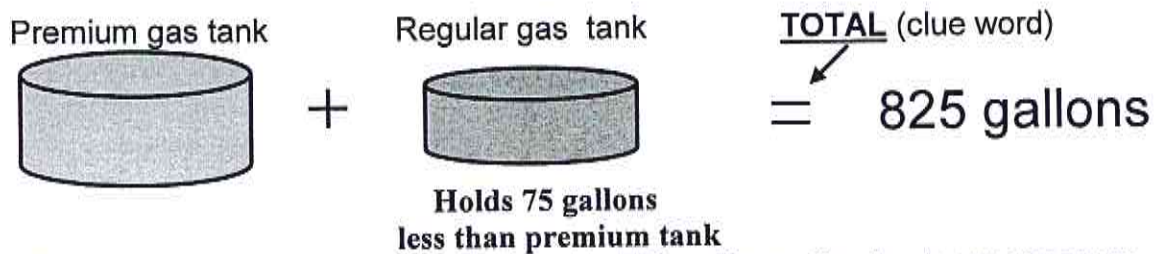
Solve the equation as done with integers.

WORD PROBLEMS

Following these steps will help you get through a word problem:

1. Draw a picture that matches your information.
2. Put all information from the problem on your picture. (In English - Not Algebra)
If you can tell what the problem says by looking at your picture, then you have done an excellent job.
OPTION: Make a chart for all your information. (Great organizational tool.)
3. To know that you really understand the problem, try putting in a **reasonable guess** of the answer and then figure out if it is correct or incorrect. You could continue to guess at the answer until you get it correct, but eventually guessing will take up too much of your time.
4. Using **algebra** will allow us to find the correct answer without all the guessing. This means that we will use a **variable to represent the correct answer** in place of the value we were guessing. We will write an equation very similar to the work as when we were guessing. Equation clue words: TOTAL, SUM, or similar words.
5. Solve the algebraic equation. Check your answer.
6. Answer the original question by writing in sentence format.

Example: At a service station, the underground tank storing regular gas holds 75 gallons less than the tank storing premium gas. If the total storage capacity of the two tanks is 825 gallons, how much does the premium gas tank hold?



I am going to **guess** that the premium tank holds **500 gallons**. To check our guess we can calculate that the regular gas tank holds 425 gallons (75 less than the premium tank of 500 gallons). If our guess had been correct the total for the two tanks would be 825. But because 500 and 425 totals to the incorrect value of 925, we need to guess again!

Using **algebra** we can assign a variable (the correct answer) to the amount in the premium tank. The correct amount in the premium tank is **P**. Therefore the amount in the regular tank (which is 75 gallons less) is **P - 75**. Now since these answers are correct the total will be 825 gallons.

Reasoning: Amount in premium tank + Amount in regular tank = 825 gallons

Equation: $P + P - 75 = 825$

Solve: $2P - 75 = 825$ (Combine like terms)

$2P = 900$ (Add 75 to both sides of equation)

$P = 450$ (Divide both sides by 2)

Answer: The premium tank holds 450 gallons of gas.

WHAT THE WORDS MEAN

Addition:	Sum	the sum of x and 5	$x + 5$
Addition:	Plus	12 plus y	$12 + y$
Addition:	Increased by	h increased by 8	$h + 8$
Addition:	Exceeds	exceeds m by 6	$m + 6$
Addition:	Added to	7 added to m	$m + 7$
Addition:	More than	h more than 4	$4 + h$
Addition:	Greater than	3 greater than y	$y + 3$
Subtraction:	Difference	the difference of k and 23	$k - 23$
Subtraction:	Minus	13 minus x	$13 - x$
Subtraction:	Decreased by	h decreased by 8	$h - 8$
Subtraction:	Reduced by	13 reduced by r	$13 - r$
Subtraction:	Less	24 less m	$24 - m$
Subtraction:	Less than	15 less than c	$c - 15$
Subtraction:	Subtracted from	8 subtracted from r	$r - 8$
Multiplication:	Product	the product of 15 and x	$15x$
Multiplication:	Times	5 times m	$5m$
Multiplication:	Twice	twice w	$2w$
Multiplication:	Of	half of t	$\frac{1}{2}t$
Division:	Per	miles per gallon	miles/gallon
Division:	Quotient	the quotient of r and 7	$r / 7$
Division:	Divided by	4 divided by y	$4 / y$
Division:	Ratio	the ratio of z to 13	$z / 13$
Division:	Split into	5 split into n equal parts	$5 / n$
Exponent:	Square	the square of m	m^2
Exponent:	Cube	the cube of r	r^3
Equals:	Is	The sum of 4 and r is 9	$4 + r = 9$
Equals:	Result is	If you increase m by 7, the <u>result is</u> twice x	$m + 7 = 2x$
Inequality	Is not equal to	y is not equal to twice x	$y \neq 2x$
Inequality	Is less than	6 is less than r	$6 < r$
Inequality	Is greater than	m is greater than y	$m > y$
Inequality		r is less than or equal to z	$r \leq z$
Inequality		p is greater than or equal to w	$p \geq w$

GROUPING SYMBOLS

Tells you to **FIRST** try to simplify what is **inside** the grouping symbol.

- Parentheses: $4(w + 7)$ Cannot be simplified inside grouping!
The distributive property will allow you to eliminate this grouping.
 $4w + 28$ Expression without grouping symbol.

- Fraction bar: $\frac{8r - 5}{3}$ Cannot be simplified inside grouping!
The distributive property will allow you to eliminate this grouping.
 $\frac{8r}{3} - \frac{5}{3}$ Expression without grouping symbol.

- Radical: $\sqrt{2m + 49}$ Cannot be simplified inside grouping!
 $\sqrt{2m} + \sqrt{49}$ **WRONG! INCORRECT! ERROR!**

$$\sqrt{25 + 36} \quad \text{Can be simplified inside square root.}$$
$$\sqrt{61} \quad \text{CORRECT simplified form.}$$

- Absolute value: $|2x - 5|$ Cannot be simplified inside grouping!
 $|2x| - |5|$ **WRONG! INCORRECT! ERROR!**

$$\begin{array}{l} |-15 + 8| \\ |-7| \\ 7 \end{array} \quad \begin{array}{l} \text{Can be simplified inside absolute value.} \\ \\ \text{CORRECT simplified form.} \end{array}$$

NOTE: The “P” in **PEMDAS** represents “Parentheses” but it also applies to all other Grouping symbols. If there are parentheses inside of other parentheses, be sure to work from the **INSIDE** to the **OUTSIDE**.

EXPONENTS

A number in the upper right hand corner of the BASE that **TELLS** you how times to multiply the BASE number by ITSELF.

Example: 3^4 mean $3 \cdot 3 \cdot 3 \cdot 3 = 81$ (Do NOT multiply anything by 4.)

Expand 4567.89 using powers of 10 for each column:

$$4 \cdot 1000 + 5 \cdot 100 + 6 \cdot 10 + 7 \cdot 1 + 8 \cdot \frac{1}{10} + 9 \cdot \frac{1}{100}$$
$$4 \cdot 10^3 + 5 \cdot 10^2 + 6 \cdot 10^1 + 7 \cdot 10^0 + 8 \cdot 10^{-1} + 9 \cdot 10^{-2}$$

Definition: Any number to the zero power has a value of ONE (1)
Examples: $5^0 = 1$ and $x^0 = 1$

- Multiplying the same base with exponents:
 $x^5 \cdot x^3$ means $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$ means x^8
Shortcut: Add the exponents

- Power of a power:
 $(x^4)^2$ means $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$ means x^8
Shortcut: Multiply the exponents

Example:

$$(2x^2)^3$$
$$(2xx)(2xx)(2xx)$$
$$8x^6$$

Example:

$$(3x^3y^2)(5x^4y^3)$$
$$(3 \cdot 5)(x^3 \cdot x^4)(y^2 \cdot y^3)$$
$$15x^7y^5$$

Example:

$$4m^3(3m^4)^2$$
$$4m^3(3m^4)(3m^4)$$
$$(4 \cdot 3 \cdot 3)(m^3 \cdot m^4 \cdot m^4)$$
$$36m^{11}$$

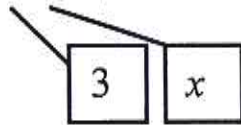
Be careful: -6^2 means “What is the opposite of 6 squared?”
6 squared is 36 and the opposite of 36 is -36.

But

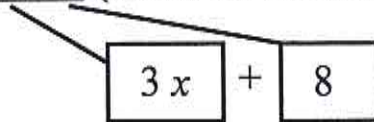
$(-6)^2$ means “What is negative 6 squared?”
 $(-6)(-6)$ is 36. •

EQUATION PARTS with instructions

Factor (number or variable(s) that are multiplied):



Term (numbers and/or variables that are added or subtracted):



Expression (One or more terms):

$$3x + 8$$

SIMPLIFY an expression using $GE(\overline{MD})(\overline{AS})$.

1. Work inside all Groupings.

(Parentheses), [Brackets], $\frac{\text{Fraction}}{\text{Bar}}$, $\sqrt{\text{Radical}}$, |Absolute value|

2. Exponents

3. Do Multiplication and Division combinations **Left to Right**.

4. Do Addition and Subtraction combinations **Left to Right**.

Equation (2 expressions that are equal to each other):

$$3x + 8 = x - 2$$

Left expression = Right expression

SOLVE an equation by doing the same thing to both expressions.

One is on the left side of equals sign and one is on the right side.

1. Simplify: $GE(\overline{MD})(\overline{AS})$ both sides of the equation.

2. Collect all variables into ONE TERM (add / subtract to both sides).

3. Isolate the variable term (add / subtract constant term to both sides).

4. Isolate the variable itself (multiply/divide coefficient to both sides).

Additional Developmental Math I Practice:

1. What is the perimeter of a rectangle that has a length of 8 cm and a width of 5 cm?
a) 40 sq. cm b) 40 cm c) 26 cm d) 26 sq. cm
2. What is the area of a square that has a side length of 12 centimeters?
a) 48 cm b) 48 sq. cm c) 144 sq. cm d) 144 cm
3. What is the area of a triangle that has a base of 5 inches and a height of 8 inches?
a) 20 inches b) 40 inches c) 20 sq. inches d) 40 sq. inches
4. What is the volume of a cube that measures 11 meters on each side?
a) 1331 meters b) 44 meters c) 1331 cu. meters d) 1331 sq. meters
5. Use order of operations to simplify: $(8 - 3 \cdot 2)^3 - 3$
a) 997 b) 27 c) 3 d) 5
6. Use order of operations to simplify: $-4 \cdot 6 - (-5)^2 + 2$
a) 3 b) -27 c) 1 d) -47
7. Use order of operations to simplify: $10 - [6 - 3(5 - 8)]$
a) 13 b) 1 c) -5 d) 19
8. Simplify: $|-9| - (-4)$
a) -13 b) -5 c) 5 d) 13
9. Simplify: $-2 + |-5 + 6|$
a) -3 b) -1 c) 9 d) 13
10. Simplify: $9x^2y - 4xy - x^2y - 6xy$
a) $8x^2y + 10xy$ b) $-2x^4y^2$ c) $8x^4y^2 - 10x^2y^2$ d) $8x^2y - 10xy$
11. Simplify: $-\frac{3}{5}z + \frac{1}{2}z$
a) $-\frac{1}{10}z$ b) $\frac{2}{3}z$ c) $-\frac{3}{10}z$ d) $\frac{1}{10}z^2$

12. Simplify: $(1.6y)(0.23z)$
 a) $1.83yz$ b) $d) 0.368yz$ c) $3.68yz$ d) $1.83 + yz$
13. Evaluate the following expression for $x = 3$ and $y = -4$: $x^2 - y^2$
 a) -2 b) 14 c) -7 d) 25
14. Evaluate the following expression for $x = \frac{2}{3}$ and $y = \frac{1}{2}$: $(x - y)(x + y)$
 a) $\frac{7}{36}$ b) $\frac{4}{3}$ c) $\frac{3}{5}$ d) $\frac{1}{9}$
15. Evaluate the following expression for $x = 0.2$ and $y = 3.4$: $x(y^2 - 4)$
 a) 223.2 b) 1.512 c) 0.56 d) 2.304
16. Solve for x : $5x - 10x = 25$
 a) $x = \frac{5}{3}$ b) $x = 5$ c) $x = -20$ d) $x = -5$
17. Solve for x : $-3x + 7 = x - 5$
 a) $x = \frac{1}{2}$ b) $x = -\frac{1}{2}$ c) $x = -3$ d) $x = 3$
18. Solve for x : $2(8x - 1) = 6(5 - x)$
 a) $x = \frac{14}{11}$ b) $x = \frac{16}{11}$ c) $x = \frac{16}{5}$ d) $x = \frac{14}{5}$
19. Solve for y : $\frac{3}{4} + y = \frac{5}{3}$
 a) $y = \frac{20}{9}$ b) $y = -2$ c) $y = \frac{11}{12}$ d) $y = \frac{29}{12}$
20. Solve for z : $\frac{6}{5}z = \frac{4}{7}$
 a) $z = -\frac{22}{35}$ b) $z = \frac{62}{35}$ c) $z = \frac{24}{35}$ d) $z = \frac{10}{21}$
21. Solve for z : $5.6 = 0.02z + 7.38$
 a) 0.2596 b) 649 c) 0.0356 d) -89
22. Solve for x : $\frac{x}{0.08} = 5.8$
 a) 5.72 b) 5.88 c) 72.5 d) 0.464

23. The formula for the perimeter of a rectangle is: $P = 2L + 2W$
Solve for 'W' when $P = 23$ and $L = 7$.
a) $W = 4.5$ b) $W = 2$ c) $W = 18.5$ d) $W = 18$
24. Best Buy is selling a television for \$1250.00. Sales tax in Orange County is 6%. Using 'P' as the amount I will have to pay for the television (including sales tax), write an algebraic equation that describes this transaction.
a) $P = (0.06)(1250)$ b) $P = 1250 \div 0.06$
c) $P = 1250 + (0.06)(1250)$ d) $P = 1250(0.94)$
25. Jeremy put \$1250 into his savings account, which pays 5% per year simple interest and left it there for 3 years. Using 'A' as the total amount that will be in the bank at the end of the 3 years, write an algebraic equation that describes this transaction.
a) $A = 1250 + 1250(0.05)(3)$ b) $A = 1250(0.05)(3)$
c) $A = 1250 - 1250(0.05)(3)$ d) $A = 1250 + (0.05)(3)$
26. Keisha is investing her money in an IRA. Initially she will be putting in \$775. Using 'C' as the additional amount invested each month, translate this problem into an algebraic expression that will show how much Keisha invested for the entire year.
a) $775 + C$ b) $775 - 12C$ c) $775C$ d) $775 + 12C$
27. Multiply and simplify where possible: $4x(5x - 2)$
a) $20x^2 - 2$ b) $20x - 8$ c) $12x$ d) $20x^2 - 8x$
28. Multiply and simplify where possible: $5x(3y + 4z - 11)$
a) $15y + 20z - 55x$ b) $-4xyz$
c) $8xy + 9xz - 6x$ d) $15xy + 20xz - 55x$
29. Multiply and simplify where possible: $xy(6x^2 - 3y^3z)$
a) $6x^3y - 3xy^4z$ b) $7x^3y - 2xy^4z$
c) $6x^2y - 3xy^3z$ d) $3x^3y^4z$
30. Multiply and simplify where possible: $(4x - 3)(6x - 7)$
a) $10x - 10$ b) $10x^2 - 46x - 10$
c) $24x^2 + 21$ d) $24x^2 - 46x + 21$

31. Multiply and simplify where possible: $\left(z + \frac{3}{4}\right)\left(z - \frac{3}{4}\right)$
- a) $z^2 - \frac{3}{4}$ b) $2z$ c) $z^2 - \frac{2}{3}$ d) $z^2 - \frac{9}{16}$
32. Multiply and simplify where possible: $(5x - z)(2x - 7z)$
- a) $10x - 37xz + 7z$ b) $10x^2 + 7z^2$
 c) $7x - 8y$ d) $10x^2 - 37xz + 7z^2$
33. Simplify: $(6x^2 - 3x - 7) + (3x^2 + 5)$
- a) $9x^2 - 3x - 2$ b) $9x^2 - 3x - 12$
 c) $3x^2 - 3x - 2$ d) $9x^2 - 3x + 12$
34. Simplify: $(z^2 - 3z + 1) - (7z^2 - 8z + 5)$
- a) $-6z^2 - 11z + 6$ b) $-6z^2 + 5z - 4$
 c) $-6z^2 - 11z - 4$ d) $-7z^2 + 5z - 4$
35. Simplify: $(z^2 + 3) + (4z - 7) - (5z^2 + z - 9)$
- a) $-4z^2 + 5z + 5$ b) $-4z^2 + 3z + 5$
 c) $-4z^2 + 5z - 13$ d) $-6z^2 + 3z - 13$

Answers:

- | | |
|-------|-------|
| 1. C | 19. C |
| 2. C | 20. D |
| 3. C | 21. D |
| 4. C | 22. D |
| 5. D | 23. A |
| 6. D | 24. C |
| 7. C | 25. A |
| 8. D | 26. D |
| 9. B | 27. D |
| 10. D | 28. D |
| 11. A | 29. A |
| 12. B | 30. D |
| 13. C | 31. D |
| 14. A | 32. D |
| 15. B | 33. A |
| 16. D | 34. B |
| 17. D | 35. B |
| 18. B | |

Part 3

Developmental Math II

MAT0028

*Previously called Beginning Algebra
MAT0024*

Note: Material from this section will be on the PERT test. If you do not know the material from this section then you will probably be placed in the Developmental Math I (MAT0018) course.

Test for Developmental Math II

1a. Simplify: $2\sqrt{18a^2x^5}$

- a. $36ax^2\sqrt{x}$
- b. $18a^2x^4\sqrt{2x}$
- c. $18ax^2\sqrt{2x}$
- d. $6ax^2\sqrt{2x}$

1b. Simplify: $6\sqrt{5} + \sqrt{6} - 2\sqrt{5}$

- a. $4\sqrt{10} + \sqrt{6}$
- b. $4\sqrt{5} + \sqrt{6}$
- c. $5\sqrt{16}$
- d. $5\sqrt{6}$

2a. Identify the greatest common factor: $3x^2y - 6xy^2$

- a. $3x^2y^2$
- b. $3xy$
- c. 3
- d. $-3xy$

2b. $m^2 - n^2$ factors into:

- a. $mn(m - n)$
- b. $(m - n)(m - n)$
- c. $(m + n)(m - n)$
- d. not factorable

2c. Identify a factor of the following trinomial: $2x^2 + 5x - 12$

- a. $(2x + 3)$
- b. $(x + 4)$
- c. $(x + 3)$
- d. $(2x - 6)$

2d. Simplify: $\frac{3x^2 - 4x - 15}{x^2 - 4x + 3}$

a. $\frac{3x - 5}{x - 1}$

b. $\frac{x + 5}{x - 1}$

c. $\frac{3x + 5}{x - 1}$

d. $\frac{3x + 5}{2x - 3}$

3a. Simplify: $\frac{x^2 y^3}{x^{-3} y^5}$

a. $\frac{x^5}{y^2}$

b. $\frac{y^2}{x^5}$

c. $\frac{x^6}{y^2}$

d. xy^3

3b. Simplify: $(5x^3 y^0)^{-2}$

a. $\frac{1}{25x^6}$

b. $10x^6$

c. $\frac{25}{x^5 y^2}$

d. $-10x^{-6}$

4a. Find the x and y intercepts and the slope of the given equation: $2x + 3y = 12$

a. $(6, 0), (0, 4), -\frac{2}{3}$

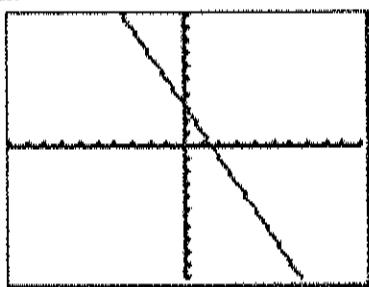
b. $(4, 0), (0, 6), \frac{3}{2}$

c. $(6, 0), (0, 4), \frac{3}{2}$

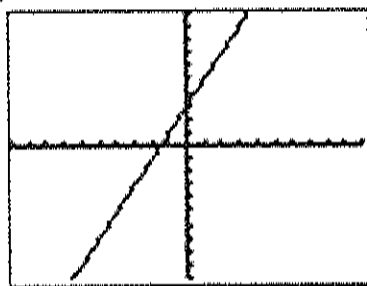
d. $(4, 0), (0, 6), -\frac{2}{3}$

4b. Match the following equation to the appropriate graph: $y = -2x + 3$
Each mark on the axes represents one (1) unit.

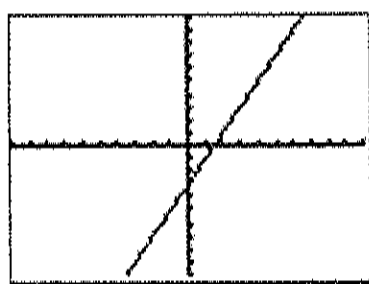
a.



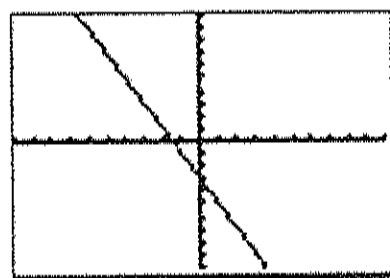
b.



c.



d.



5a. Solve for x : $\frac{3}{4}x - 5 = 3$

a. $x = \frac{3}{32}$

b. $x = -\frac{8}{3}$

c. $x = \frac{32}{3}$

d. $x = -\frac{3}{2}$

5b. Solve for x: $ax + by = c$

a. $\frac{a}{c - by}$

b. $\frac{by - c}{a}$

c. $\frac{c - by}{a}$

d. $\frac{c + by}{a}$

5c. Solve for x: $4x^2 + 4x - 35 = 0$

a. $x = -\frac{7}{2}, x = -\frac{5}{2}$

b. $x = -\frac{7}{2}, x = \frac{5}{2}$

c. $x = \frac{2}{7}, x = \frac{5}{2}$

d. $x = \frac{7}{2}, x = -\frac{5}{2}$

5d. Solve for x: $5x - 8 < 11x - 2$

a. $x < 1$

b. $x > 1$

c. $x < -1$

d. $x > -1$

6a. Convert to scientific notation: 0.000125

a. 1.25×10^4

b. 1.25×10^{-4}

c. 0.125×10^{-3}

d. 1.25×10^6

6b. Convert to standard form: 4.52×10^5

a. 0.0000452

b. 452000

c. 45200000

d. 4.52000

Developmental Math II Test Answers:

- 1a. d
- 1b. b
- 2a. b
- 2b. c
- 2c. b
- 2d. c
- 3a. a
- 3b. a
- 4a. a
- 4b. a
- 5a. c
- 5b. c
- 5c. b
- 5d. d
- 6a. b
- 6b. b

DM-II Question #1: Square Roots

You need to know from memory all the following perfect squares:

$$\sqrt{4}, \sqrt{9}, \sqrt{16}, \sqrt{25}, \sqrt{36}, \sqrt{49}, \sqrt{64}, \sqrt{81}, \sqrt{100}, \sqrt{121}, \sqrt{144}, \dots$$

$$\text{and } \sqrt{a^2}, \sqrt{b^2}, \sqrt{c^2}, \sqrt{d^2}, \dots, \sqrt{x^2}, \sqrt{y^2}, \sqrt{z^2}$$

Simplifying a square root means to find all perfect square factors that are located inside the square root symbol (radical) and remove them.

Simplify:	$\sqrt{18}$	Example
	$\sqrt{9 \cdot 2}$	Factor out the perfect square 9
	$\sqrt{9} \cdot \sqrt{2}$	Separate the square root of 9 from the square root of 2
	$3\sqrt{2}$	Replace the square root of 9 with the value 3

Simplify:	$\sqrt{75x^3}$	Example
	$\sqrt{25 \cdot 3 \cdot x^2 \cdot x}$	Factor out the perfect squares 25 and x^2
	$\sqrt{25} \cdot \sqrt{x^2} \cdot \sqrt{3x}$	Separate the perfect squares of 25 and x^2
	$5x\sqrt{3x}$	Replace the square roots of 25 and x^2 with 5 and x

Simplify:	$\sqrt{48x^3y^6}$	Example
	$\sqrt{16 \cdot 3 \cdot x^2x \cdot y^2y^2y^2}$	Factor out the perfect squares: 16, x^2 , y^2 , y^2 , y^2
	$\sqrt{16}\sqrt{x^2}\sqrt{y^2}\sqrt{y^2}\sqrt{y^2}\sqrt{3x}$	Separate the perfect squares of 16, x^2 , y^2 , y^2 , y^2
	$4xyyy\sqrt{3x}$	Replace the square roots of 16, x^2 , y^2 with 4, x, y
	$4xy^3\sqrt{3x}$	Simplify the y's using exponents

Like terms have the same value inside the square root (radical):

Like radicals

$$\sqrt{2} \quad \text{and} \quad 3\sqrt{2}$$

$$2\sqrt{5} \quad \text{and} \quad 7\sqrt{5}$$

$$4\sqrt{3m} \quad \text{and} \quad 7\sqrt{3m}$$

Unlike radicals

$$\sqrt{2} \quad \text{and} \quad \sqrt{11}$$

$$3\sqrt{7} \quad \text{and} \quad 7\sqrt{3}$$

$$6\sqrt{5r} \quad \text{and} \quad 7\sqrt{5h}$$

Example:	$5\sqrt{7} + 2\sqrt{7} - 4\sqrt{7}$	simplifies to:	$3\sqrt{7}$
Example:	$5\sqrt{3} + 2\sqrt{5x} + 6\sqrt{3} + 4\sqrt{5x}$	simplifies to:	$11\sqrt{3} + 6\sqrt{5x}$

Simplify: $\sqrt{45x} + \sqrt{125x}$ Example
 $\sqrt{9 \cdot 5x} + \sqrt{25 \cdot 5x}$ Factor out the perfect squares: 9 and 25
 $\sqrt{9} \cdot \sqrt{5x} + \sqrt{25} \cdot \sqrt{5x}$ Separate the perfect squares: 9 and 25
 $3\sqrt{5x} + 5\sqrt{5x}$ Replace the square roots of 9 and 25 with 3 and 5
 $8\sqrt{5x}$ Because these are like terms they can be added

Example: $6\sqrt{3}(\sqrt{5} + 2\sqrt{7})$ Because we have unlike terms inside the parentheses
 $6\sqrt{3} \cdot \sqrt{5} + 6\sqrt{3} \cdot 2\sqrt{7}$ we need to distribute to simplify this example.
 $6\sqrt{15} + 12\sqrt{21}$ **Multiplication (and division) allows us to combine any square roots.**

$\sqrt{9+16} = \sqrt{25} = 5$ Correct

$\sqrt{9+16} \neq \sqrt{9} + \sqrt{16}$ Watch out!

Summary: $\sqrt{A+B} \neq \sqrt{A} + \sqrt{B}$

This is not correct.

<u>Simplify</u>	<u>Answer</u>
1. $\sqrt{25m^2}$	5m
2. $\sqrt{18r^3}$	$3r\sqrt{2r}$
3. $\sqrt{180k^8}$	$6k^4\sqrt{5}$
4. $\sqrt{200g^7z^4}$	$10g^3z^2\sqrt{2g}$
5. $3y\sqrt{147d^{12}y^5z}$	$21d^6y^3\sqrt{3yz}$
6. $\sqrt{64} + \sqrt{121}$	19
7. $\sqrt{54} - \sqrt{150}$	$-2\sqrt{6}$
8. $\sqrt{49x^3} + \sqrt{81x}$	$7x\sqrt{x} + 9\sqrt{x}$
9. $\sqrt{24} + 5\sqrt{96}$	$22\sqrt{6}$
10. $k\sqrt{7x^2} + 4x\sqrt{63k^2}$	$13kx\sqrt{7}$
11. $\sqrt{3}(\sqrt{7} + \sqrt{2})$	$\sqrt{21} + \sqrt{6}$
12. $4\sqrt{5}(6\sqrt{7} + 9\sqrt{2})$	$24\sqrt{35} + 36\sqrt{10}$
13. $2\sqrt{5r}(\sqrt{3r} + 8\sqrt{2h})$	$2r\sqrt{15} + 16\sqrt{10hr}$
14. $3\sqrt{8}(\sqrt{8} + 2\sqrt{18})$	96

DM-II Question #2:

Factoring

A **FACTOR** is a number, letter, term, or polynomial that is **multiplied**. Factoring requires that you put (parentheses) into your expression.

Step 1: Look for factor(s) that are common to ALL terms.
Common factors are written on the outside of the parentheses.
Inside the parentheses is what is left after removing the common factor(s) from each term.

Example: Factor completely the following polynomial.

$$\begin{array}{ll} 15x^5 + 25x^2 & \text{Binomial expression} \\ 5 \cdot 3 \cdot x^2 \cdot x^3 + 5 \cdot 5 \cdot x^2 & 5 \text{ and } x^2 \text{ are common factors} \\ 5x^2(3x^3 + 5) & \text{Completely factored polynomial} \end{array}$$

Completely means there are **NO MORE** common factors.

Note: Factor completely also means Find Greatest Common Factor

Next we should look for factors that are binomials:

Step 2: $x^2 \pm \square \pm \square$ **Step 3:** $x^2 \pm \square \pm 12$

$\swarrow \quad \searrow$ $\swarrow \quad \searrow$
 $(x \quad \quad)(x \quad \quad)$ $(x \quad 3)(x \quad 4)$

**** Try factors that are closest together in value for your first try.**

Step 4: $x^2 \pm \square \pm 12$ The last sign tells you to add or subtract the **Inside \pm Outside** terms = middle term.

$$\begin{array}{ccccc} (x & 3) & & (x & 4) \\ \vdots & & & \vdots & \\ & 3x & \leftarrow & & \\ & 4x & \leftarrow & & \end{array}$$

If your answer equals the middle term, your information is correct.

Note: If there is no middle term, then it has a value of zero (0).

Step 5: Assign positive or negative values to the Inside term (3x) and the outside term (4x) so the combined value will equal the middle term. Put these signs into the appropriate binomial.

DM-II Questions #2: Factoring Examples

Factor completely: This means that you are expected to **LOOK** for any common factors (GCF) **BEFORE** looking for possible binomial factors. Factoring problems may have either or both of these types.

Directions: Factor completely:

Example: $10x^2 + 11x - 6$ Trinomial
Step 1: No common factors Always look for common factor(s) **first**.
Step 2: $(5x \quad)(2x \quad)$ What 2 factors equal $10x^2$ (**F** in FOIL)?


Try factors that are closest together in value first !!!!

$10x$ and x are possibilities, but are not used as often.

Step 3: $(5x \quad 3)(2x \quad 2)$ What 2 factors equal 6 (**L** in FOIL)?

Since 3 and 2 are closer in value than 6 and 1 we have made a good choice. But the $2x$ and 2 together have a common factor (we did common factors in step 1 so this should not occur), therefore the factors should be switched so there are no common factor(s) in either binomial.

Step 4: $(5x \quad 2)(2x \quad 3)$ Better choice to factor this polynomial.
 $10x^2 \quad 15x \quad 4x \quad 6$ Outside term ($15x$) and Inside term ($4x$).
Are these Outside ($15x$) and Inside ($4x$) terms correct? **YES!!!!**

 The second sign (subtraction) indicates that if we subtract our **Outside term ($15x$)** and **Inside term ($4x$)** to total **$11x$** our factors would be correct.
Since $15x - 4x = 11x$ we know our factors were placed correctly!

Step 5: Assign appropriate signs: The combined total of the Outside ($15x$) and Inside ($4x$) terms will be a **POSITIVE 11**. This would require the $15x$ be positive and $4x$ be negative.

Factored completely: $(5x - 2)(2x + 3)$

Example: $36y^3 - 66y^2 + 18y$ Factor completely:

Step 1: $6y(6y^2 - 11y + 3)$ Factor out $6y$ (GCF).

Step 2: $6y(3y \quad)(2y \quad)$ Factor $6y^2$ (**F** in FOIL).

Step 3: $6y(3y \quad 1)(2y \quad 3)$ Factor 3 (**L** in FOIL).

Step 4: Outside term ($9y$) + Inside term ($2y$) = $11y$ This is **CORRECT!!**

Step 5: Assign appropriate signs: The combined total of the Outside ($9y$) and Inside ($2y$) terms will be a **NEGATIVE 11**. This would require both the $9y$ and $2y$ be negative.

Factored completely: $6y(3y - 1)(2y - 3)$

DM-II Question #2: Reducing (Simplifying) Rational Expressions

STEPS FOR SIMPLIFYING:

1. Put parentheses around any fraction bar grouping to remind you it is **ALL** or **NOTHING**
2. Factor out all **COMMON** factors
3. Factor out all **BINOMIAL** factors
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero)
5. Reduce (Watch for **GROUPING** symbols)

Example: $\frac{2x^2 + 6x}{x^2 + 5x + 6}$

Step 1: $\frac{(2x^2 + 6x)}{(x^2 + 5x + 6)}$ This grouping has common factors of 2 and x
This grouping has 2 binomial factors of (x + 3) and (x + 2)

Step 2&3: $\frac{2x(x + 3)}{(x + 3)(x + 2)}$

Step 4: Restrictions: $x \neq \{-3, -2\}$ [Remember that you cannot divide by zero.]

Step 5: $\frac{2x\cancel{(x + 3)}}{\cancel{(x + 3)}(x + 2)}$ (x + 3) is a binomial factor of the numerator and the denominator. **REDUCE** to 1.

Simplified: $\frac{2x}{(x + 2)}$

<u>Questions</u>	<u>Answers</u>
1. $\frac{15x}{3x^2}$	$\frac{5}{x}$
2. $\frac{2a - 10}{2}$	$a - 5$
3. $\frac{8k - 16}{k^2 - 4}$	$\frac{8}{k + 2}$
4. $\frac{x^2 - 7x + 12}{2x^2 - 5x - 12}$	$\frac{x - 3}{2x + 3}$

DM-II Question #3: Exponent Shortcuts

When simplifying rational expressions (fractions) with variable factors, the object is to have the variable appear only once in the expression with a positive exponent.

1. $(x^a)(x^b) = x^{a+b}$ When multiplying with same base, add exponents.

Examples: $(x^6)(x^9) = x^{15}$
 $(r^5)(r^{-3}) = r^2$
 $(p^{-4})(p^{-2})(p^{13}) = p^7$

2. $\frac{x^a}{x^b} = x^{a-b}$ When dividing with the same base, subtract the exponents.

Examples: $\frac{r^9}{r^4} = r^5$ or $\frac{w^{11}}{w^8} = w^3$ or $\frac{h^5}{h^{-3}} = h^8$

3. $(x^a)^b = x^{ab}$ When raising a power to a power, multiply the exponents.

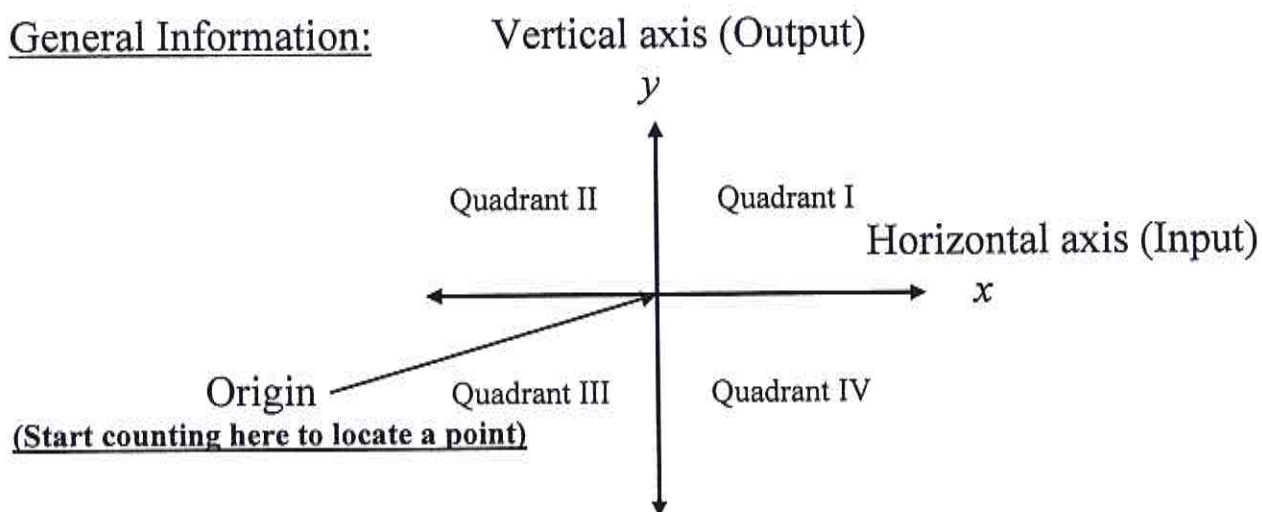
Examples: $(g^3)^4 = g^{12}$
 $(x^{-5})^{-2} = x^{10}$

4. $x^{-a} = \frac{1}{x^a}$ or $\frac{1}{x^{-a}} = x^a$ To remove a negative exponent, you will need to find the reciprocal of what the negative exponent is located on.

Examples: $n^{-4} = \frac{1}{n^4}$ or $\frac{1}{v^{-7}} = v^7$ or $\frac{z^5 z^{-2}}{z^{-4} z^9} = \frac{z^5 z^4}{z^9 z^2} = \frac{z^9}{z^{11}} = \frac{1}{z^2}$

DM-II Question #4: GRAPHING – Linear Equations

General Information:



$\begin{array}{l} \text{ } \rightarrow x\text{-coordinate} \\ \text{ } \rightarrow y\text{-coordinate} \end{array}$ (Go to the right 7 units from the origin)
(7, -5) Ordered pair (Go down 5 units from the origin)
(x-value, y-value)

A point (represented by an ordered pair) is located by starting at the ORIGIN (where the axes cross) then move horizontally according to the x -coordinate and then vertically according to the y -coordinate.

It is assumed that you will count by ONES on a graph, if not, you **MUST** note your scale on the axes. The two axes do NOT have to have the same scale.

Graphing a linear equation: [x and y are to the 1st power]
The graph will always be a straight line with arrows on both ends.

Example:

$$y = 3x - 4$$

Table of values

x	y
5	11
1	-1
-3	-13
0	-4

Most values are found by picking a value for x (that will fix on your graph paper) then put this value into your equation to calculate the y -value.

For best results attempt to use positive, negative, and zero values.

Linear equations formats:

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Point-slope: $y = m(x - h) + k$

Graphing information:

$$\text{Slope}(m) = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Point = (x_1, y_1)

y - intercept ($x = 0$)

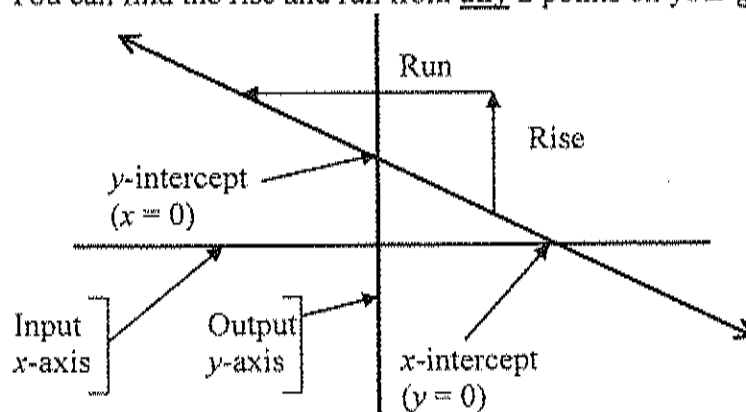
x - intercept ($y = 0$)

Y (answer) in terms of **X** (work)

Output in terms of **Input**

Graphs

Slope: You can find the rise and run from any 2 points on your graph!



To put a line on your graph paper:

Step 1: Establish a point on your graph.

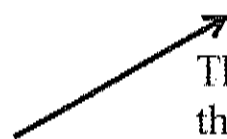
Step 2: Use the slope to establish a second point.

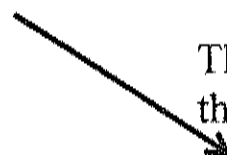
Step 3: Draw your line through the 2 points.

DM-II Question #4: Linear Graph, Slope, x-intercept, y-intercept

A linear equation can give specific information (slope and y-intercept) about a graph when written in the slope-intercept format.

Slope – Intercept form of a linear equation: $y = mx + b$

 This is a positive slope ($m > 0$) because you see the graph **rise** as you read from left to right.

 This is a negative slope ($m < 0$) because you see the graph **fall** as you read from left to right.

y – intercept (b) is the point where your graph crosses the y -axis.

Any form of a linear equation:

To find x -intercept: Set $y = 0$ (or cover up the y -term) and solve for x .

To find y -intercept: Set $x = 0$ (or cover up the x -term) and solve for y .

To find slope: Rewrite in Slope – Intercept format and read m -value.

<u>Question</u>	<u>Answer</u>
1. $y = 4x + 7$ Find slope & y -intercept	Slope = 4 ; y -intercept = 7
2. $y = -5x - 3$ Find slope & y -intercept	Slope = -5 ; y -intercept = -3
3. $y = \frac{3}{4}x + 8$ Find slope & y -intercept	Slope = $\frac{3}{4}$; y -intercept = 8
4. $2x + 3y = 5$ Find slope & y -intercept	Slope = $-\frac{2}{3}$; y -intercept = $\frac{5}{3}$
5. $4x + 5y = 3$ Find x and y -intercepts	$(\frac{3}{4}, 0)$ and $(0, \frac{3}{5})$
6. $5x - 2y = 10$ Find x and y -intercepts	$(2, 0)$ and $(0, -5)$
7. $-6x + 5y = 2$ Find the slope	Slope = $\frac{6}{5}$

DM-II Question #5: Solving

Linear Equation: [One variable – Answer is a number]

1. Simplify both sides of the equation individually.

*This means to apply **GEMDAS** to both sides.*

2. Collect the variable you are solving from both sides into 1 term.

This may be done by adding or subtracting the variable term to both sides.

3. Isolate the term containing the variable you are solving for.

This may be done by adding or subtracting the constant term (the one without a variable in it) to both sides.

4. Isolate the variable you are solving for.

This may be done by dividing by the coefficient on both sides.

Literal Equation: [Multiple variables – Answer has variables in it]

Similar to Linear Equation above, but uses more than one variable.

Inequality: [One variable – Answer is a number]

1. Simplify both sides of the equation individually.

*This means to apply **GEMDAS** to both sides.*

2. Collect the variable you are solving from both sides into 1 term.

This may be done by adding or subtracting the variable term to both sides.

3. Isolate the term containing the variable you are solving for.

This may be done by adding or subtracting the constant term (the one without a variable in it) to both sides.

4. Isolate the variable you are solving for.

This may be done by dividing by the coefficient on both sides.

- If **YOU** multiplied or divided **both sides by a negative** number, you must **reverse** the inequality.

Example: $2x + 7(x - 2) > 13x - 2 + 8$

Step 1: $2x + 7x - 14 > 13x + 6$

$$9x - 14 > 13x + 6$$

Step 2: $9x - 14 - 13x > 13x + 6 - 13x$

$$-4x - 14 > 6$$

Step 3: $-4x - 14 + 14 > 6 + 14$

$$-4x > 20$$

Step 4: $\frac{-4x}{-4} < \frac{20}{-4}$ *Divided by a negative no.*

$$x < -5$$
 Reverse the inequality

Quadratic Equation: [Has x^2 and two answers]

- General form: $ax^2 + bx + c = 0$
- Square root method: Use if there is no "x" term [$b = 0$]

Example: $4x^2 - 100 = 0$

$$4x^2 = 100 \quad (\text{Add 100 to both sides})$$

$$\frac{4x^2}{4} = \frac{100}{4} \quad (\text{Divide both sides by 4})$$

$$x^2 = 25 \quad (\text{Simplify each side})$$

$$\sqrt{x^2} = \sqrt{25} \quad (\text{Take square root of both sides})$$

$$x = \pm 5 \quad (\text{Simplify each side})$$

Answers: $x = \{-5, 5\}$

- Factoring method: Set equation = 0 [General form]

Example: $x^2 + 4x - 12 = 0$ [$c \neq 0$]]

$$(x + 6)(x - 2) = 0$$

$$x + 6 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -6 \quad x = 2$$

Answers: $x = \{-6, 2\}$

Example: $x^2 + 5x = 0$ [$c = 0$]

$$x(x + 5) = 0$$

$$x = 0 \quad \text{or} \quad x + 5 = 0$$

$$x = 0 \quad x = -5$$

Answers: $x = \{-5, 0\}$

- Quadratic formula method: Set equation = 0 [General form]

Ex: $2x^2 + 5x - 7 = 0$ [$a = 2, b = 5, c = -7$]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot (-7)}}{2 \cdot 2}$$

$$x = \frac{-5 \pm \sqrt{25 + 56}}{4}$$

$$x = \frac{-5 \pm \sqrt{81}}{4}$$

$$x = \frac{-5 \pm 9}{4} \quad \text{therefore:}$$

$$x = \frac{-5 + 9}{4} \quad \text{and} \quad x = \frac{-5 - 9}{4}$$

$$x = 1 \quad \text{and} \quad x = \frac{-7}{2}$$

DM-II Question #6: Scientific Notation

Definition of scientific notation:

A between number 1 **and** 10 multiplied by a power of 10.

Example: 5.3407×10^{12} or 3.0218×10^{-7}

Logic: If we multiply a number by 10 one or more times, the value of the number will get larger.

Rule: Positive powers of 10 will move the decimal point to the RIGHT!

Example: 7.345×10^5
734500

Example: 3045900000
 3.0459×10^9

Logic: If we divide a number by 10 one or more times, the value of the number will get smaller.

Rule: Negative powers of 10 (same as dividing by 10) will move the decimal point to the LEFT.

Example: 2.37×10^{-7}
0.000000237

Example: 0.000000000431
 4.31×10^{-10}

Questions

Put the following scientific numbers into standard form:

- | | |
|-----------------------------|--------------------|
| 1. 3.4×10^4 | 34,000 |
| 2. 6.908×10^9 | 6,908,000,000 |
| 3. 1.0377×10^{13} | 10,377,000,000,000 |
| 4. 7.23×10^{-5} | 0.0000723 |
| 5. 2.0256×10^{-11} | 0.000000000020256 |

Put the following standard form numbers into scientific notation:

- | | |
|-----------------------|------------------------|
| 6. 1,000,000 | 1.0×10^6 |
| 7. 67,800,000,000,000 | 6.78×10^{13} |
| 8. 0.00000034 | 3.4×10^{-7} |
| 9. 0.00004055 | 4.055×10^{-5} |
| 10. 0.0000000000081 | 8.1×10^{-11} |

Setting up Word Problems - Using 1 Variable

1. Draw a picture that matches your information.
2. Put all information from the problem on your picture.
Option: Make a chart for all your information. (Great organizational tool.)
3. Look for relationships among the information. (Definition, formula, logical conclusion)
4. If lost, try assigning a **reasonable guess** to the question and calculate if the guess is correct or incorrect. Guessing will give a better understanding of the information.
5. Assign a variable (the correct value) to the unknown and write an equation that should be similar to the information that you were guessing in the previous step.
6. Solve equation.
7. Answer the original question in sentence form.

Example: The length of a rectangle is 24 feet more than the width. The perimeter of the rectangle is 88 feet. Find the length of the rectangle.

Length = 24 feet more than the width ($W + 24$)

Width



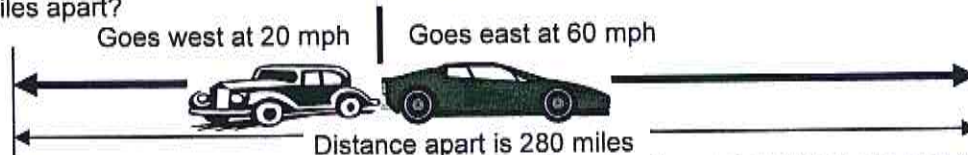
Perimeter: Distance all the way around the outside of the rectangle is 88 feet.

Definition: Perimeter is equal to the sum of the four sides.

Variable: If I knew the width, then I could use that info to find the length. Therefore I will assign the variable W to represent the correct width and $W + 24$ to represent the correct length.

Setup:
$$\begin{array}{rclclcl} \text{Perimeter} & = & \text{Width} & + & \text{Length} & + & \text{Width} & + & \text{Length} \\ 88 & = & W & + & (W + 24) & + & W & + & (W + 24) \end{array}$$

Example: Two cars leave the same point at the same time traveling in opposite directions. One car travels west at 20 miles per hour and the other travels east at 60 miles per hour. In how many hours will they be 280 miles apart?



Definition: Distance traveled is equal to the product of the rate of speed and time elapsed. ($D = R \cdot T$)

Logic: The distance **apart** is the total mileage of both cars because they are going in **opposite** directions. We also total the rates for the same reason. But travel time is unaffected by direction.

Time in this problem will be the **same** for both cars because they start and stop at the same time.

Variable: Use T as the correct number of hours of travel time for the cars.

Setup:
$$\begin{array}{rcl} \text{Distance} & = & \text{Rate of speed} \cdot \text{Time} \\ 280 \text{ miles apart} & = & 80 \text{ miles apart after each hour (80 mph)} \cdot T \text{ hours traveling} \end{array}$$

Example: Find the amount of money now necessary to be invested at 5% simple interest to yield \$320 interest in 4 years. (Yield means the amount of money produced at end of time in bank.)



Principal (\$ you put in bank) \rightarrow 5% Rate of interest \rightarrow 4 years later = \$320 total interest


Definition: Principal times Rate of interest per year times Number of years equals Total interest.


Variable: P is the correct amount of money we need to put into the bank.

Setup:
$$\begin{array}{rcl} \text{Principal} \cdot \text{Rate} \cdot \text{Time} & = & \text{Interest (yield)} \\ P \cdot 0.05 \cdot 4 & = & \$320 \end{array}$$


Geometry


- Point – Basic geometric figure.


 Segment – Two points and all those in between. Part of a line.

 Ray – Starts at a point and extends forever in one direction.

 Line – Extends forever in opposite directions.

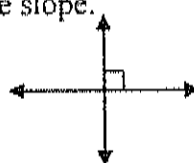
 Angle – Measurement (in degrees) around a point.
Formed by two rays with a common endpoint (vertex).

 Congruent – Two geometric figures with same measurement.

 Parallel lines – Do not intersect. Have the same slope.

 Perpendicular lines – Meet at right angles.

Their slopes are opposites and reciprocals.



Single angles:

Acute angle: Between 0° and 90° .

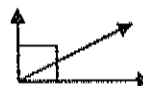
Right angle: 90° . Formed by perpendicular lines. (Do not assume.)

Obtuse angle: Between 90° – 180° .

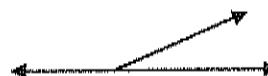
Straight angle: 180° If it looks like a straight angle, then assume it is.

Pairs of angles:

Complementary angles: Any two angles that total 90° .

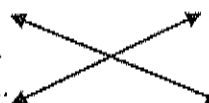


Supplementary angles: Any two angles that total 180° .



Vertical angles: Opposite angles formed by two intersecting lines.

Measurements of vertical angles are always equal.



Triangle: A 3-sided polygon with 3 angles that total 180° .



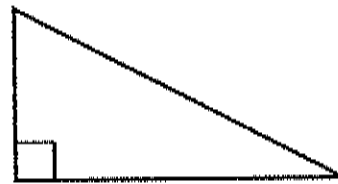
Equilateral

$3 \cong \text{sides} \ \& \ 3 \cong \text{angles}$



Isosceles

$2 \cong \text{sides} \ \& \ 2 \cong \text{angles}$



Right triangle

$$a^2 + b^2 = c^2$$

Similar triangles

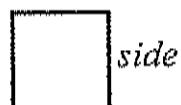
Corresponding angles are congruent (\cong).

Corresponding sides are proportional.



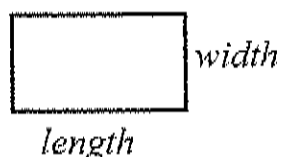
Perimeter (units): *Distance around the outside of polygon.*
Area (square units): *Number of squares you can count inside of a polygon.*

Square



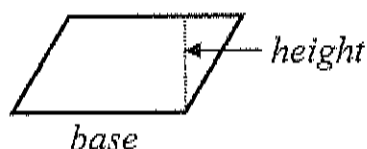
Perimeter = $4s$ units
 Area = s^2 square units

Rectangle



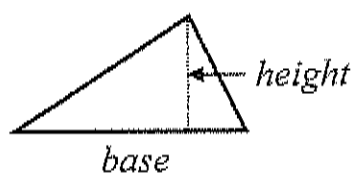
Perimeter = $2l + 2w$ units
 Area = lw square units

Parallelogram



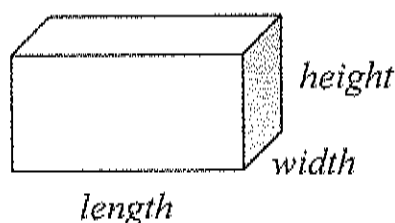
Perimeter = Add all 4 sides together.
 Area = bh square units

Triangle



Perimeter = Add all 3 sides together.
 Area = $\frac{1}{2}bh$ square units

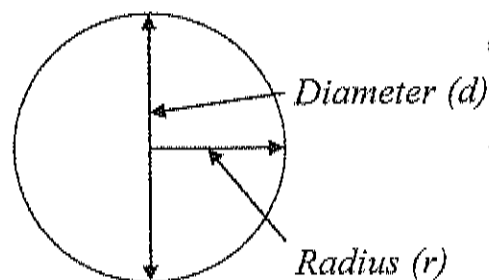
Rectangular
solid



Surface area = $2lw + 2lh + 2wh$ sq. units

Volume = lwh cubic units
Number of cubes in the box!

Circle



Circumference = $2\pi r$ or πd units

Area = πr^2 square units

Pi (π) is the number (≈ 3.14) of diameters in the circumference.

RATIONAL EXPRESSIONS – Multiplication/Division

STEPS FOR MULTIPLYING (or dividing) 2 FRACTIONS:

1. Put parentheses around any fraction bar grouping to remind you it is **ALL** or **NOTHING**.
 2. Factor out all **COMMON** factors.
 3. Factor out all **BINOMIAL** factors.
 4. Find all restrictions (Values of the variable that will cause the denominator to equal zero).
 5. If division: Find the reciprocal of the fraction **AFTER** the division sign and put in mult. sign.
 6. Multiply your numerators and then multiply your denominators.
- NOTE: Do NOT actually multiply groupings, but show to be multiplied.
7. Reduce.

Multiplication example:

$$\frac{x^2 + x}{2x - 8} \cdot \frac{12}{x^2 + 3x + 2}$$

Step 1:
$$\frac{(x^2 + x)}{(2x - 8)} \cdot \frac{12}{(x^2 + 3x + 2)}$$

Step 2&3:
$$\frac{x(x+1)}{2(x-4)} \cdot \frac{12}{(x+2)(x+1)}$$

Step 4: Restrictions: $x \neq \{-2, -1, 4\}$

Step 5:

Step 6:
$$\frac{12x(x+1)}{2(x-4)(x+2)(x+1)}$$

Step 7: The factors **2** and **(X + 1)** are common to the numerator and denominator. **REDUCE** to 1.

Simplified:
$$\frac{6x}{(x-4)(x+2)}$$

Division example:

$$\frac{x^2 + x}{2x - 8} \div \frac{3x}{6x^2 - 12x - 48}$$

$$\frac{(x^2 + x)}{(2x - 8)} \div \frac{3x}{(6x^2 - 12x - 48)}$$

$$\frac{x(x+1)}{2(x-4)} \div \frac{3x}{6(x-4)(x+2)}$$

Restrictions: $x \neq \{-2, 0, 4\}$

$$\frac{x(x+1)}{2(x-4)} \cdot \frac{6(x-4)(x+2)}{3x}$$

$$\frac{6x(x+1)(x-4)(x+2)}{2 \cdot 3x(x-4)}$$

The factors **6**, **X**, and **(X - 4)** are common to the numerator and denominator. **REDUCE** to 1.

$$(x+1)(x+2)$$

RATIONAL EXPRESSIONS – Addition/Subtraction

STEPS FOR Adding or Subtracting 2 FRACTIONS:

1. Put parentheses around any fraction bar grouping to remind you it is **ALL** or **NOTHING**.
2. Factor out all **COMMON** factors.
3. Factor out all **BINOMIAL** factors.
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero).
5. Find a **COMMON** denominator (If you already have one go to step 6).
 - A. Write all factors of **FIRST** denominator.
 - B. Multiply by **ANY OTHER** factors of second denominator.
 - C. Multiply by **ANY OTHER** factors of subsequent denominators.
 - D. Use identity of multiplication to get your fractions to this common denominator.
6. Add / Subtract your numerators. (Keep your same common denominator).
(Remember that subtraction will **CHANGE** the sign of **EVERY TERM** being subtracted).
7. Simplify **NUMERATOR** (NOT your denominator).
8. Factor numerator.
9. Reduce.

Example: $\frac{2x-5}{6x+9} - \frac{4}{2x^2+3x} + \frac{1}{x}$

Step 1: $\frac{(2x-5)}{(6x+9)} - \frac{4}{(2x^2+3x)} + \frac{1}{x}$

Step 2&3: $\frac{(2x-5)}{3(2x+3)} - \frac{4}{x(2x+3)} + \frac{1}{x}$

Step 4: Restrictions: $x \neq \{0, -\frac{3}{2}\}$

Step 5A,B,C: Lowest Common Denominator: $3x(2x+3)$

Step 5D: $\frac{(2x-5)}{3(2x+3)} \cdot \frac{x}{x} - \frac{4}{x(2x+3)} \cdot \frac{3}{3} + \frac{1}{x} \cdot \frac{3}{3} \cdot \frac{(2x+3)}{(2x+3)}$

Step 5D: $\frac{x(2x-5)}{3x(2x+3)} - \frac{12}{3x(2x+3)} + \frac{3(2x+3)}{3x(2x+3)}$ (Simplified by multiplication)

Step 6: $\frac{x(2x-5)-12+3(2x+3)}{3x(2x+3)}$

Step 7: $\frac{2x^2+x-3}{3x(2x+3)}$

Step 8: $\frac{(x-1)(2x+3)}{3x(2x+3)}$

Step 9: $\frac{x-1}{3x}$

Signed Numbers Review

Simplify:

1. $-4 + (-8)$

2. $-20 + 13$

3. $9 - 15$

4. $-5 - 7$

5. $11 - (-8)$

6. $-3 - (-15)$

7. $(-12)(6)$

8. $(-6)(-8)$

9. $24 \div (-8)$

10. $9 - (-4) - 6$

11. $-11 + 6 \cdot (-9)$

12. $-12 \div 3 \cdot 2 - (-4)$

13. $-5 - 4^2$

14. -3^4

15. $-5^2 - (-2)^4$

16. $-(-12 \div 4 \cdot 3)$

17. $-\frac{3}{4} + \frac{3}{8}$

18. $\frac{1}{6} - \frac{3}{5}$

19. $-\frac{2}{5} + \left(-\frac{1}{2}\right)$

20. $-\frac{2}{3} - \frac{5}{6}$

21. $\frac{7}{10} - \left(-\frac{5}{7}\right)$

22. $\left(-\frac{2}{3}\right)\left(\frac{5}{6}\right)$

23. $\left(-\frac{3}{8}\right)\left(-\frac{5}{8}\right)$

24. $\frac{5}{6} \div \left(-\frac{2}{3}\right)$

25. $-\frac{9}{10} \div \left(-\frac{10}{9}\right)$

26. $4 - \frac{7}{8}$

27. $\frac{7}{8} - 4$

28. $\left(-\frac{2}{3}\right)^3$

29. $-\left(\frac{2}{3}\right)^2$

30. $\frac{5}{6} - \left(-\frac{3}{5}\right)^2$

31. $\frac{3}{4} - \frac{1}{2} + \frac{2}{3}$

32. $\frac{5}{6} \cdot \left(-\frac{3}{4}\right) \div \left(-\frac{1}{4}\right)$

33. $\left(-\frac{3}{5}\right)^3$

34. $-\left(\frac{5}{4}\right)^2 + \frac{1}{2}$

35. $23.4 - 123$

36. $56.21 + (-13.5)$

37. $-3.304 - 2.6$

38. $-0.58 + (-3.07)$

39. $(3.12)(-2.05)$

40. $(-5.5)(-33)$

41. $9.01(-.008)$

42. $-25.615 \div 0.5$

43. $-135.27 \div (-0.09)$

44. $15 \div (-0.8)$

45. $(-0.45) \div (-5)(0.8)$

46. $-1.6 - 2.5(-0.9)$

47. $(1.2 - 5)(-0.24 + 3)$

48. $-(2.6)^2$

49. $(-7.5 - 0.89)^2$

50. $(-0.3)^4 - (1.2)^3$

Answers:

1. -12

2. -7

3. -6

4. -12

5. 19

6. 12

7. -72

8. 48

9. -3

10. 7

11. -65

12. -4

13. -21

14. -81

15. -41

16. 9

17. $-\frac{3}{8}$

18. $-\frac{13}{30}$

19. $-\frac{9}{10}$

20. $-\frac{3}{2}$

21. $\frac{99}{70}$

22. $-\frac{5}{9}$

23. $\frac{15}{64}$

24. $-\frac{5}{4}$

25. $\frac{81}{100}$

26. $\frac{25}{8}$ or $3\frac{1}{8}$

27. $-\frac{25}{8}$

28. $-\frac{8}{27}$

29. $-\frac{4}{9}$

30. $\frac{71}{150}$

31. $\frac{11}{12}$

32. $\frac{5}{2}$

33. $-\frac{27}{125}$

34. $-\frac{17}{16}$

35. -99.6

36. 42.71

37. -5.904

38. -3.65

39. -6.396

40. 1.815

41. -0.07208

42. -51.23

43. 1503

44. -18.75

45. 0.072

46. 0.65

47. -10.488

48. -6.76

49. 70.3921

50. -1.7199

Linear Equation Review

Solve:

1. $x - (-5) = -8$

2. $x - \frac{3}{4} = \frac{1}{2}$

3. $y - 2.56 = -4.7$

4. $-8x = 12$

5. $\frac{2}{3}x = 5$

6. $1.1z = 34.21$

7. $-0.4x = 6.3$

8. $-5 = -3x$

9. $\frac{3}{8} = \frac{2}{3}y$

10. $-9.6 = 0.3x$

11. $5z - 11z = -2$

12. $\frac{1}{4}y = \frac{3}{8} + \frac{1}{2}$

13. $-2.5x + 2.2x = -17.61$

14. $-8x + 2 = x - 11$

15. $-9 - (-12) = -3y - y$

16. $\frac{5}{6}x + \frac{1}{2} = -\frac{1}{3}x$

17. $-5.61 - 0.1y = 0.1y$

18. $-6z + 9 = 8z - (-2)$

19. $-\frac{4}{5}x - \frac{2}{3} = x - \frac{3}{4}$

20. $\frac{-2x}{1.4} = 3.4 - 0.8$

21. $\frac{z}{5} + \frac{3z}{5} = -\frac{2}{3}$

22. $5 - y = -8$

23. $6.5x - 4.8x + 0.38 = -2$

24. $-11z - 9 - (-4) = -5z - z$

25. $\frac{3}{x} = 8$

26. $-2(3x - 5) = -4$

27. $6(3y - 2) = -(-3y + 5)$

28. $\frac{3}{5}\left(\frac{4}{5}x - \frac{1}{2}\right) = -\frac{3}{4}$

29. $2.66 = 1.4(0.5z - z)$

30. $-2(-3y + 5) - y = -9y + 4$

31. $5(-3x - 7) = -3(9x - 4)$

32. $-\frac{2}{3}\left(\frac{1}{5}x - \frac{3}{4}\right) = \frac{1}{2}\left(-\frac{5}{6}x + \frac{2}{5}\right)$

33. $\frac{x}{-2} - 5 = \frac{3}{4}x$

34. $\frac{1.4z}{-5} - 3.4 = 2.4(0.8z - 3)$

Answers:

1. $x = -13$

2. $x = \frac{5}{4}$

3. $y = -2.14$

4. $x = -\frac{3}{2}$

5. $x = \frac{15}{2}$

6. $z = 31.1$

7. $x = -15.75$

8. $x = \frac{5}{3}$

9. $y = \frac{9}{16}$

10. $x = -32$

11. $z = \frac{1}{3}$

12. $y = \frac{7}{2}$

13. $x = 58.7$

14. $x = \frac{13}{9}$

15. $y = -\frac{3}{4}$

16. $x = -\frac{3}{7}$

17. $y = -28.05$

18. $z = \frac{1}{2}$

19. $x = \frac{5}{108}$

20. $x = -1.82$

21. $z = -\frac{5}{6}$

22. $y = 13$

23. $x = -1.4$

24. $z = -1$

25. $x = \frac{3}{8}$

26. $x = \frac{7}{3}$

27. $y = \frac{7}{15}$

28. $x = -\frac{15}{16}$

29. $z = -3.8$

30. $y = 1$

31. $x = \frac{47}{12}$

32. $x = -\frac{18}{17}$

33. $x = -4$

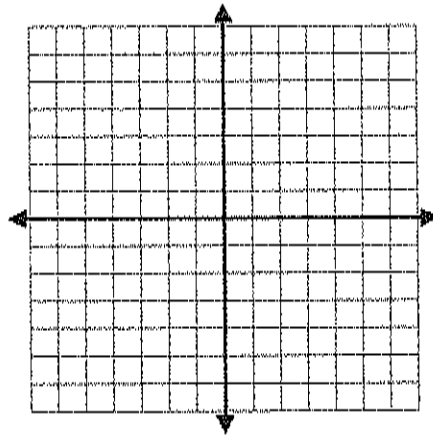
34. $z = \frac{19}{11}$

Graphing Review

1. Mark and label the following points on the graph below.

$A = (3,5)$; $B = (-4,-6)$; $C = (7,-2)$; $D = (0,4)$

Draw and label both axes. Show appropriate scale.

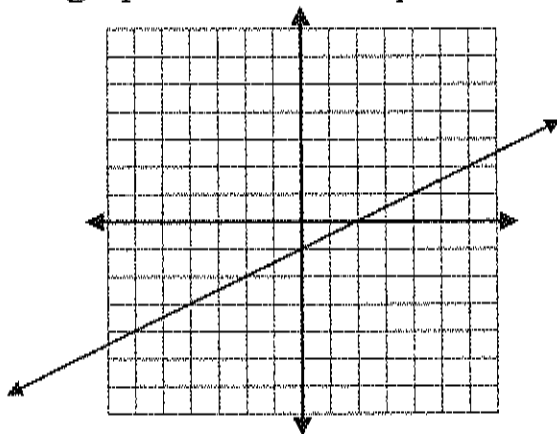


2. Fill in the chart with appropriate values that satisfy this equation:

$y = 3x - 5$

x	y
3	
	10
-2	
	0
0	
	-17

3. Fill in the chart with 6 appropriate integer values that satisfy this graph: Units are equal to one.

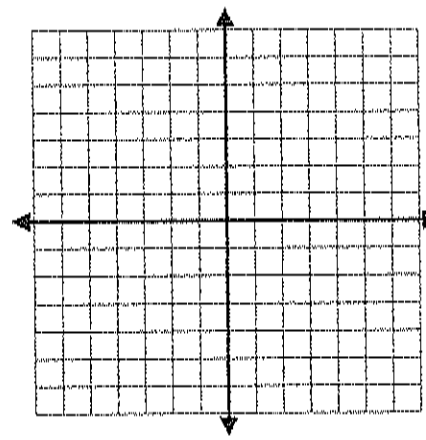
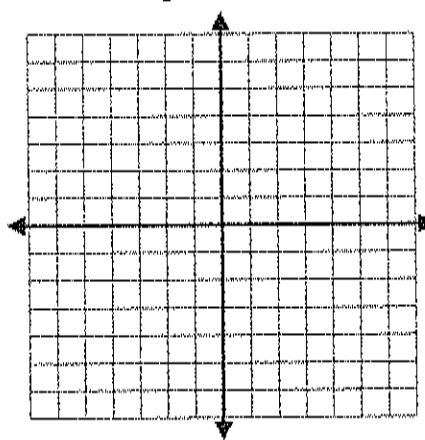
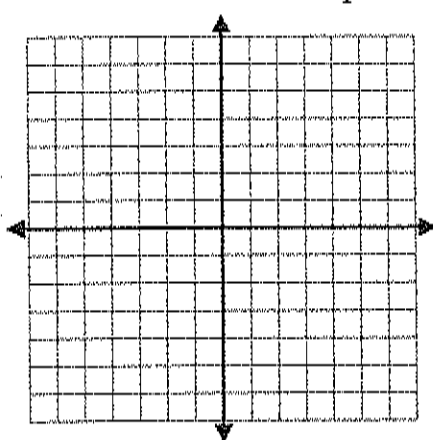


x	y

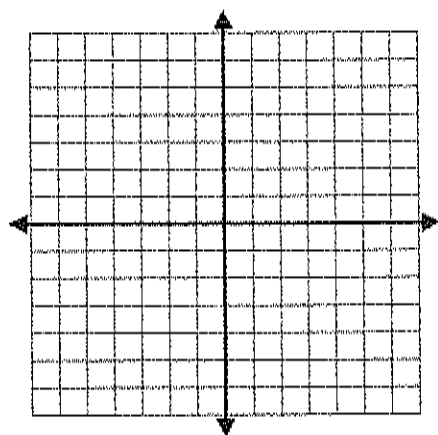
4. Find the x and y - intercept: (Write each as an ordered pair.)

$2x + 3y = 12$	x - intercept: _____	y - intercept: _____
$x - 2y = 10$	x - intercept: _____	y - intercept: _____
$-3x - 5y = 7$	x - intercept: _____	y - intercept: _____
$y = 4x - 1$	x - intercept: _____	y - intercept: _____
$x = 6$	x - intercept: _____	y - intercept: _____

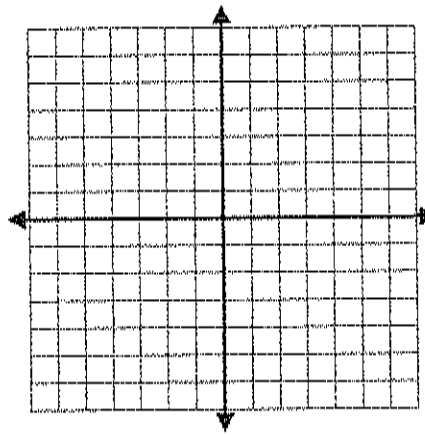
5. A. Graph below: Slope = $\frac{2}{5}$ y - intercept = 2
 B. Graph below: Slope = -2 y - intercept = 0
 C. Graph below: Slope = $\frac{5}{3}$ y - intercept = -5



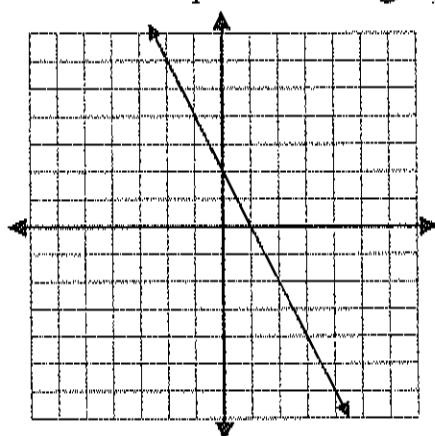
6. Graph below: $x = 5$



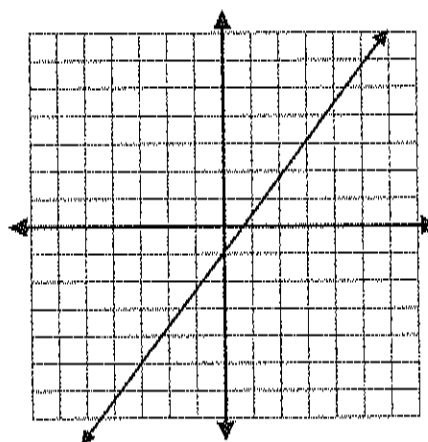
Graph below: $y = -4$



7. Find the slope of each graph:



Slope = _____



Slope = _____

Graph Answers: All spaces will represent one unit unless otherwise noted.

- Point A will be 3 spaces to the right and 5 spaces up from the origin.
Point B will be 4 spaces to the left and 6 spaces down from the origin.
Point C will be 7 spaces to the right and 2 spaces down from the origin.
Point D will be 4 spaces up from the origin on the y-axis.

2.

x	y
3	4
5	10
-2	-11
$\frac{2}{3}$	0
0	-5
-4	-17

3.

x	y
0	-1
2	0
6	2
-4	-3
4	1
-6	-4

4. Find the x and y - intercept: (Write each as an ordered pair.)

$$2x + 3y = 12$$

$$x - \text{intercept: } (6, 0)$$

$$y - \text{intercept: } (0, 4)$$

$$x - 2y = 10$$

$$x - \text{intercept: } (10, 0)$$

$$y - \text{intercept: } (0, -5)$$

$$-3x - 5y = 7$$

$$x - \text{intercept: } (-\frac{7}{3}, 0)$$

$$y - \text{intercept: } (0, -\frac{7}{5})$$

$$y = 4x - 1$$

$$x - \text{intercept: } (\frac{1}{4}, 0)$$

$$y - \text{intercept: } (0, -1)$$

$$x = 6$$

$$x - \text{intercept: } (6, 0)$$

$$y - \text{intercept: none}$$

5A. This graph should go through the y-axis at 2 and through the x-axis at -5.

5B. This graph should go through the origin (0,0) and points (1, -2) and (-1, 2).

5C. This graph should go through the y-axis at -5 and through the x-axis at 3.

6. The graph of $x = 5$ is a vertical line that goes through the x-axis at 5.

The graph of $y = -4$ is a horizontal line that goes through the y-axis at -4.

7. The slope of the first graph is -2. The slope of the second graph is $\frac{4}{3}$.

Polynomial Review

1. $3x + 2x - x$
2. $4y + 8x - 6y - 3x$
3. $x - y + 4z - 5z + 8y$
4. $5x^2 + 6x - 2x^2 - 10x$
5. $5y^2 - x^2 + y^2 - x^2$
6. $8z^3 + 5z^3$
7. $x^4 + 4x^3 + 5x^4$
8. $-4x^2 - 5x^2 + 9x - x$
9. $-5y - 4y - y + x$
10. $-6y + 3z - 9z + y - z$
11. $(2x)(6y)$
12. $(7x)(3x)$
13. $(-4x)(8x)$
14. $(3x)(9y)(4z)$
15. $(-7x)(2x^2)$
16. $(5x^2)(5y^2)$
17. $(-x^2)(7x^3)$
18. $(2z^2)(-3z^2)(z^2)$
19. $(-8x^4)(-6y^3)(-z^2)$
20. $(-3z^3)(6y^3)(-4z^4)(-y)$
21. $3(2x + 5)$
22. $-5(8y + 2)$
23. $-3(-4z + 2)$
24. $x(4x + 5)$
25. $-5y(x + 3y + 6z)$
26. $(2x + 5)(3x + 4)$
27. $(5x - 4)(2x - 3)$
28. $(x + y)(2x + 5y)$
29. $(-2x - 5)(-4x - 3)$
30. $(-x + 5y)(-3x + 2)$
31. $(7y - 2)^2$
32. $(-5y + z)^2$
33. $(2x - 3z^2)^2$
34. $2z(-3 + 4y^2 - 7z)$
35. $(2x + 5)(-4x + 3y - 1)$
36. $\frac{2xy}{6x}$
37. $\frac{4x^2y}{2x}$
38. $\frac{-12y^3z^2}{6xyz}$
39. $\frac{-3x^3z^4}{-12x^5z}$
40. $\frac{2x + 3y}{6xy}$
41. $\frac{2x - 5z^4}{-xz}$
42. $\frac{-3x^2 + 5y^5 - xy}{7xy}$
43. $4(3y + 5) + 2(-3y + 4)$
44. $3x(y + 7) - 2y(3x - 4)$
45. $2(z^2 + 1) + 4(z + 3z^2)$
46. $-3(x - y) + y(2x + 5)$
47. Factor: $x^2 + 7x + 12$
48. Factor: $z^2 - 8z + 15$
49. Factor: $3x^2 - 17x + 10$
50. Factor: $2x^2 + 11x + 12$

Answers:

1. $4x$
2. $5x - 2y$
3. $x + 7y - z$
4. $3x^2 - 4x$
5. $-2x^2 + 6y^2$
6. $13z^3$
7. $6x^4 + 4x^3$
8. $-9x^2 + 8x$
9. $x - 10y$
10. $-5y - 7z$
11. $12xy$
12. $21x^2$
13. $-32x^2$
14. $108xyz$
15. $-14x^3$
16. $25x^2y^2$
17. $-7x^5$
18. $-6z^6$
19. $-48x^4y^3z^2$
20. $-72y^4z^7$
21. $6x + 15$
22. $-40y - 10$
23. $12z - 6$
24. $4x^2 + 5x$
25. $-5xy - 15y^2 - 30yz$
26. $6x^2 + 23x + 20$
27. $10x^2 - 23x + 12$
28. $2x^2 + 7xy + 5y^2$
29. $8x^2 + 26x + 15$
30. $3x^2 - 15xy - 2x + 10y$
31. $49y^2 - 28y + 4$
32. $25y^2 - 10yz + z^2$
33. $4x^2 - 12xz^2 + 9z^4$
34. $-6z + 8y^2z - 14z^2$
35. $-8x^2 + 6xy - 22x + 15y - 5$
36. $\frac{y}{3}$
37. $2xy$
38. $\frac{-2y^2z}{x}$
39. $\frac{z^3}{4x^2}$
40. $\frac{1}{3y} + \frac{1}{2x}$
41. $-\frac{2}{z} + \frac{5z^3}{x}$
42. $-\frac{3x}{7y} + \frac{5y^4}{7x} - \frac{1}{7}$
43. $6y + 28$
44. $21x - 3xy + 8y$
45. $14z^2 + 4z + 2$
46. $-3x + 2xy + 8y$
47. $(x + 3)(x + 4)$
48. $(z - 5)(z - 3)$
49. $(3x - 2)(x - 5)$
50. $(2x + 3)(x + 4)$

Practice: Variety of topics

1. Simplify: $24 \div 4 \cdot 3 - 6 \cdot 5$
a. -28 b. -12 c. 60 d. -20
2. Simplify: $20 - (-2)^2 \div (7 - 9) \cdot 13$
a. 46 b. -104 c. 286 d. $262/13$
3. Simplify: $|-7| + |17| - |7|$
a. 31 b. -17 c. -3 d. 17
4. Simplify: $-3(2x - 5) - 2(4x + 3)$
a. $14x - 9$ b. $-2x + 21$ c. $-14x + 9$ d. $-14x + 21$
5. Evaluate the given expression when $x = -4$ and $y = -1$: $3x^2 - xy + 2y^2$
a. 54 b. 46 c. -32 d. -42
6. Solve for z : $5(2z - 7) = -2z + 7$
a. $z = \frac{7}{2}$ b. $z = \frac{21}{4}$ c. $z = -\frac{7}{2}$ d. $z = -\frac{7}{3}$
7. Solve: $\frac{5}{2}x - \frac{1}{3} = -\frac{1}{2}$
a. $x = -\frac{5}{12}$ b. $x = -\frac{25}{12}$ c. $x = -\frac{1}{15}$ d. $x = \frac{1}{3}$
8. Solve for W : $P = 2L + 2W$
a. $W = P - L$ b. $W = \frac{1}{2}P - L$
c. $W = \frac{1}{2}P + L$ d. $W = 2P - L$

9. Twice the sum of a number and 8 is the same as the difference of 3 and the number. Find the equation that could be used to find the number, x .

a. $2x + 8 = x - 3$ b. $2(x + 8) = x - 3$

c. $2(x + 8) = 3 - x$ d. $2x + 8 = 3 - x$

10. Two cars start from the same place at the same time. One car travels west at 70 miles per hour and another travels east at 20 miles per hour. In how many hours will they be 250 miles apart?

a. $50/9$ hours b. 225 hours c. $25/9$ hours d. 5 hours

11. Identify the proportion listed below that solves this problem.

It will take 13 people to pick 112 watermelons in a day. How many people will it take to pick 200 watermelons in a day?

a. $\frac{112}{13} = \frac{200}{x}$ b. $\frac{13}{112} = \frac{200}{x}$ c. $\frac{112}{13} = \frac{x}{200}$ d. $\frac{13}{x} = \frac{200}{112}$

12. Simplify: $\frac{(3x^7y^7)^2}{x^9}$

a. $9x^{23}y^{14}$ b. $3x^5y^{14}$ c. $3x^{23}y^{14}$ d. $9x^5y^{14}$

13. Simplify: $\frac{x^{-3}y^{-2}}{x^2y^5}$

a. x^5y^7 b. $\frac{x^5}{y^7}$ c. $\frac{1}{x^5y^7}$ d. $\frac{y^7}{x^5}$

14. Simplify: $\left(\frac{x^{-3}y^0}{t^4}\right)^{-2}$

a. $\frac{1}{x^5y^2t^2}$ b. x^6t^8 c. $\frac{1}{x^5t^2}$ d. $\frac{x^6t^8}{y^2}$

15. Convert to scientific notation: 0.000678

- a. 6.78×10^{-4} b. 6.78×10^4 c. 67.8×10^{-5} d. 0.678×10^{-3}

16. Simplify: $(7x^2 - 5x + 7) - (x^2 - 4x - 9)$

- a. $6x^2 - x + 16$ b. $6x^2 - 9x - 2$
c. $6x^4 - x^2 + 16$ d. $6x^2 - x - 2$

17. Simplify: $2x(6x^2 + 7z)$

- a. $12x^2 + 14xz$ b. $26x^3z$ c. $8x^3 + 9xz$ d. $12x^3 + 14xz$

18. Simplify: $(2x - 9)(6x - 3)$

- a. $12x^2 + 48x + 27$ b. $12x^2 - 60x + 27$
c. $12x^2 - 48x - 27$ d. $12x^2 - 60x - 27$

19. Factor completely: $6x^2y^4 - 18x^3y + 12xy$

- a. $6xy(xy^3 - 3x^2 + 2)$ b. $xy(6xy^3 - 18x^2 + 12)$
c. $6xy(4xy^3 - x^2)$ d. $3xy(2xy^3 - 6x^2 + 4)$

20. Factor completely: $16x^2 - 25y^2$

- a. $(4x - 5y)(4x + 5y)$ b. $(4x - 5y)^2$
c. $(4x + 5y)^2$ d. $(8x - 5y)(2x + 5y)$

21. Factor completely: $3x^2 - 6x - 4xy + 8y$

- a. $(x + 2)(3x + 4y)$ b. $(x - 2)(3x - 4y)$
c. $(x + 2)(3x - 4y)$ d. $(x - 2)(3x + 4y)$

22. Identify a factor of this trinomial: $6x^2 - 17x + 12$

- a. $(2x + 3)$ b. $(3x + 4)$ c. $(3x - 4)$ d. $(6x + 1)$

23. Simplify: $\frac{3x^2 - 16x - 12}{x^2 - 7x + 6}$

a. $\frac{x+2}{x-1}$ b. $\frac{3x+2}{2x-3}$ c. $\frac{3x-2}{x-1}$ d. $\frac{3x+2}{x-1}$

24. Solve: $x^2 - 10x + 16 = 0$

a. $x = -8, x = -2$ b. $x = 8, x = 2$ c. $x = 4, x = -4$ d. $x = 16, x = 1$

25. Solve: $35x^2 + 24x + 4 = 0$

a. $x = -\frac{2}{7}, x = -\frac{2}{5}$ b. $x = -\frac{2}{7}, x = \frac{2}{5}$ c. $x = \frac{7}{2}, x = -\frac{2}{5}$ d. $x = \frac{2}{7}, x = \frac{2}{5}$

26. Assuming the variable represents a non-negative number, simplify completely:

$3\sqrt{4z^3w^6}$

a. $12w^3z\sqrt{z}$ b. $6w^3z\sqrt{z}$ c. $6w^3\sqrt{z^3}$ d. $12w^3z\sqrt{z^3}$

27. Simplify completely: $\sqrt{3}(2\sqrt{2} + \sqrt{5})$

a. $2\sqrt{5} + \sqrt{8}$ b. $2\sqrt{6} + \sqrt{15}$ c. $2\sqrt{21}$ d. $6\sqrt{6} + \sqrt{15}$

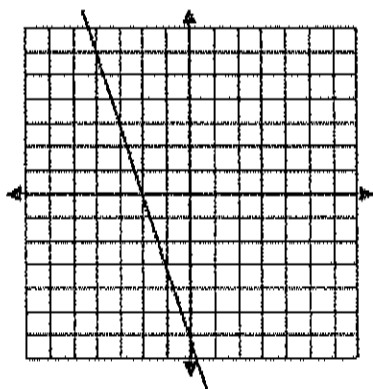
28. Solve: $8(7x + 3) > 20$

a. $x < -\frac{1}{14}$ b. $x > \frac{3}{5}$ c. $x > -60$ d. $x > -\frac{1}{14}$

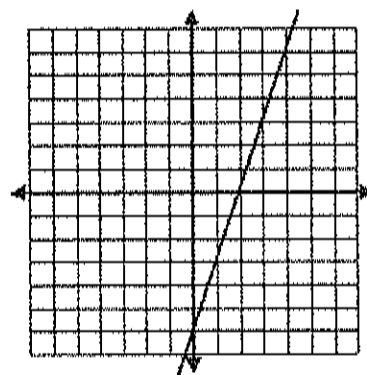
29. Find the x-intercept for: $4x - 3y = 20$

a. $(0, 5)$ b. $\left(0, \frac{-20}{3}\right)$ c. $\left(\frac{-20}{3}, 0\right)$ d. $(5, 0)$

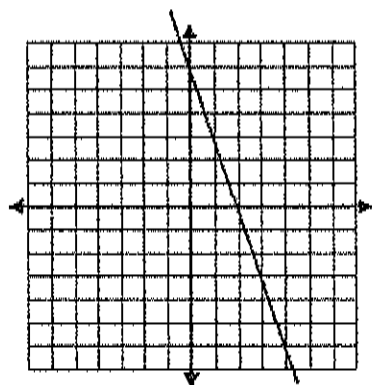
30. Find the graph that best matches the given linear equation: $y = 3x - 6$



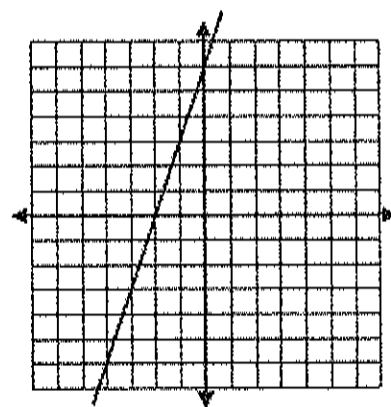
a.



b.



c.



d.

Answers:

1. b

2. a

3. d

4. c

5. b

6. a

7. c

8. b

9. c

10. c

11. a

12. d

13. c

14. b

15. a

16. a

17. d

18. b

19. a

20. a

21. b

22. c

23. d

24. b

25. a

26. b

27. b

28. d

29. d

30. b

Part 4

Intermediate Algebra

Note: This section will also be on the PERT test. If you know all this material then your score should show that you are ready for College Algebra.

Test for Intermediate Algebra

1a. Solve for x : $2 < 3x - 5 < 4$

a. $\frac{7}{3} < x < 3$

b. $-1 < x < -2$

c. $-\frac{7}{3} < x < 3$

d. $-3 < x < \frac{7}{3}$

2a. Solve the system of equations:
$$\begin{aligned} 2a + b &= 5 \\ a - b &= 1 \end{aligned}$$

a. $(-2, 1)$

b. $(2, 1)$

c. $(2, -1)$

d. $(-2, -1)$

2b. Solve the system of equations:
$$\begin{aligned} 3x + 5y &= 3 \\ x &= 8 - 4y \end{aligned}$$

a. $(-4, 3)$

b. $(3, -4)$

c. $(4, 3)$

d. $(-3, -4)$

2c. Solve the system of equations:

$$6x + 7y = 9$$

$$8x + 9y = 11$$

- a. $(2, 3)$
- b. $(-2, -3)$
- c. $(3, -2)$
- d. $(-2, 3)$

3a. Simplify: $\frac{x^2 + 9x + 8}{x^2 - 3x - 4}$

a. $\frac{x+1}{x-4}$

b. $\frac{x+8}{x+1}$

c. $\frac{x+8}{x-4}$

d. $\frac{x-8}{x+4}$

3b. Simplify: $\frac{y^2 + 10y + 25}{y^2 - 9} \cdot \frac{y^2 + 3y}{y + 5}$

a. $\frac{y(y+5)}{y+3}$

b. $\frac{y(y+5)}{y-3}$

c. $\frac{y+5}{y-3}$

d. $\frac{y^2 - 5y}{y+3}$

3c. Add and simplify: $\frac{9}{x+1} + \frac{3}{x}$

a. $\frac{12x+3}{x(x+1)}$

b. $\frac{12}{x(x+1)}$

c. $\frac{12}{2x+1}$

d. $\frac{15}{x+1}$

3d. Simplify: $\frac{x^2 - x}{x^2 - 3x + 2} \div \frac{x^2 - 1}{x^2 - 2x + 1}$

a. $\frac{x}{(x-2)(x+1)}$

b. $\frac{x(x+1)}{(x-2)(x-1)}$

c. $\frac{(x-2)(x+1)}{x(x-1)}$

d. $\frac{x(x-1)}{(x-2)(x+1)}$

3e. Simplify: $\frac{x}{x^2 + 9x + 20} - \frac{4}{x^2 + 7x + 12}$

a. $\frac{(x+5)(x+3)}{x-5}$

b. $\frac{x+5}{(x-5)(x-3)}$

c. $\frac{x-5}{(x+5)(x+3)}$

d. $\frac{x-5}{(x+3)(x+4)(x+5)}$

3f. Solve: $\frac{x-1}{x^2-2x-3} + \frac{x+2}{x^2-9} = \frac{2x+5}{x^2+4x+3}$

a. $x = -\frac{7}{3}$

b. $x = \frac{3}{7}$

c. $x = -\frac{3}{7}$

d. $x = \frac{7}{3}$

4a. $f(x) = 4x^2 - x + 3$ find $f(-2)$

a. 17

b. 21

c. -11

d. -15

4b. $f(x) = -2x^3 + 5$ find x if $f(x) = -49$

a. 2

b. 3

c. 4

d. -2

4c. Given $f(x) = 3x^2 - 2x$, find $f(a+1)$

a. $3a^2 - 2a + 1$

b. $3a^2 + 4a + 1$

c. $3a^2 - 2a + 2$

d. $3a^2 - 4a + 4$

5a. Simplify (put in rationalized form):

$$\frac{3}{\sqrt{5}}$$

a. $\frac{3\sqrt{5}}{5}$

b. $\frac{\sqrt{5}}{3}$

c. 15

d. $\sqrt{15}$

5b. Simplify (put in rationalized form):

$$\frac{8}{\sqrt{x} + 2}$$

a. $4\sqrt{x}$

b. $\frac{\sqrt{x}}{16}$

c. $\frac{8\sqrt{x} - 16}{x - 4}$

d. $\frac{8x - 16}{x + 4}$

6a. Write a linear equation in standard form that crosses (4, -3) and (-2, 9):

a. $x - y = -5$

b. $y = 2x - 5$

c. $2x + y = 5$

d. $2x - y = 5$

6b. Write a linear equation in slope-intercept form that is perpendicular to $y = 3x - 4$ crossing $(3, 6)$:

a. $y = -\frac{1}{3}x + 7$

b. $y + \frac{1}{3}x = 7$

c. $x + 3y = 21$

d. $x = -3y + 21$

Intermediate Algebra Test Answers:

1a. a

2a. b

2b. a

2c. d

3a. c

3b. b

3c. a

3d. d

3e. c

3f. a

4a. b

4b. b

4c. b

5a. a

5b. c

6a. c

6b. a

IA Question #1: **Solving Linear Inequalities**

Solve for x : $2 < 3x - 5 < 4$

When solving a compound inequality we want to isolate x in the middle of the inequality.

Remember that we must do the same thing to **all three sections** of the inequality.

$$1 < 2x + 5 < 17 \quad \text{Example}$$

$$1 - 5 < 2x + 5 - 5 < 17 - 5 \quad \text{Subtract 5 from all three sections}$$

$$-4 < 2x < 12 \quad \text{Simplify}$$

$$\frac{-4}{2} < \frac{2x}{2} < \frac{12}{2} \quad \text{Divide all three sections by 2}$$

$$-2 < x < 6 \quad \text{Simplify}$$

Note: If we multiply or divide an inequality **by a negative number** you must change the direction of the inequality.

$$-2 < -x < 3 \quad \text{Example}$$

$$\frac{-2}{-1} > \frac{-x}{-1} > \frac{3}{-1} \quad \text{Divide all 3 sections by -1}$$

Remember to reverse the direction of the inequality sign!

$$2 > x > -3 \quad \text{Simplify}$$

$$-3 < x < 2 \quad \text{Rewrite the inequality starting with smaller value.}$$

Questions

1. $3 < 2x - 3 < 5$

2. $-2 < 3x + 1 < 1$

3. $-1 < 2x + 3 < 5$

4. $4 < -2x + 3 < 6$

Answers

$$3 < x < 4$$

$$-1 < x < 0$$

$$-2 < x < 1$$

$$-\frac{3}{2} < x < -\frac{1}{2}$$

IA Question #2: SOLVE A SYSTEM OF EQUATIONS by ELIMINATION

Eliminate a variable by adding the 2 equations:

1. Line up the variables.
2. If adding the 2 equations does not give a value of zero to one of the variables, then multiply either or both equations so that zero will be one of the totals when you add them.
3. Add the 2 equations.
4. Solve the new equation.
5. Substitute your value back into either of the original equations and solve for the other variable.
6. Write your answer as an ordered pair.

Example 1:

$$2x + 5y = 12$$

$$4x - 5y = 6$$

$$6x = 18$$

$$x = 3$$

$$2(3) + 5y = 12$$

$$6 + 5y = 12$$

$$5y = 6$$

$$y = 6/5$$

$$(3, 6/5)$$

Variable sum = 0

Given equation

Given equation

Add the 2 equations

Simplify

Substitution

Simplify

Simplify

Simplify

Answers

Example 2:

$$3x - 8y = -6$$

$$-3x + 5y = 15$$

$$-3y = 9$$

$$y = -3$$

$$3x - 8(-3) = -6$$

$$3x + 24 = -6$$

$$3x = -30$$

$$x = -10$$

$$(-10, -3)$$

Example 3: Variable sum $\neq 0$:

Choosing what to multiply by is the same idea as finding a common denominator. Remember to make one term positive and one negative.

$$4x - 3y = -8 \Rightarrow$$

$$5(4x - 3y) = 5(\Rightarrow)$$

$$3x + 5y = -6 \Rightarrow$$

$$3(3x + 5y) = 3(\Rightarrow)$$

Add the 2 equations

Simplify

$$20x - 15y = -40$$

$$9x + 15y = -18$$

$$29x = -58$$

$$x = -2$$

Substitute into either of the original equations

Simplify

Simplify

Simplify

$$3(-2) + 5y = -6$$

$$-6 + 5y = -6$$

$$5y = 0$$

$$y = 0$$

Answer: $(-2, 0)$


IA Question #2: SOLVE A SYSTEM OF EQUATIONS by SUBSTITUTION

Eliminate a variable by substituting one equation into the other.

LOOK for $x = \text{something}$ or $y = \text{something}$.

1. Solve either equation for one of the variables.
2. Substitute this equation into the other one. This will leave you with one equation with only one variable.
3. Solve the new equation.
4. Substitute your value back into either of the original equations and solve for the other variable.
5. Write your answer as an ordered pair.

Example 1:

$$\begin{array}{l} y = x - 8 \\ 3x + y = 4 \end{array}$$


$$\begin{array}{l} 3x + (x - 8) = 4 \\ 3x + x - 8 = 4 \\ 4x - 8 = 4 \\ 4x = 12 \\ x = 3 \end{array}$$

$$\begin{array}{l} y = (3) - 8 \\ y = -5 \end{array}$$

(3, -5)

x or $y = \text{something}$

Given equation

Given equation

Substitution

Simplify

Simplify

Simplify

Simplify


Substitution

Simplify

Answer

Example 2:

$$4x + 5y = 11$$

$$x = 9 + 5y$$


$$\begin{array}{l} 4(9 + 5y) + 5y = 11 \\ 36 + 20y + 5y = 11 \\ 36 + 25y = 11 \\ 25y = -25 \\ y = -1 \end{array}$$

$$\begin{array}{l} x = 9 + 5(-1) \\ x = 4 \end{array}$$

(4, -1)

Note:

If you do not have an equation with $x = \text{something}$ or $y = \text{something}$:

Option 1: Solve one of the equations for ' x ' or ' y ' and then use substitution as shown in examples above.

Option 2: Line up the variables and solve by elimination as shown in examples on the previous page.

IA Question #3a: RATIONAL EXPRESSIONS – Simplifying a Fraction

STEPS FOR SIMPLIFYING ONE FRACTION:

1. Put parentheses around any fraction bar grouping to remind you it is **ALL** or **NOTHING**.
2. Factor out all **COMMON** factors.
3. Factor out all **BINOMIAL** factors.
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero).
5. Reduce (Watch for **GROUPING** symbols).

Example: $\frac{2x^2 + 6x}{x^2 + 5x + 6}$

This grouping has common factors of 2 and x.

Step 1: $\frac{(2x^2 + 6x)}{(x^2 + 5x + 6)}$

This grouping has 2 binomial factors of $(x + 3)$ and $(x + 2)$.

Step 2&3: $\frac{2x(x + 3)}{(x + 3)(x + 2)}$

Step 4: Restrictions: $x \neq \{-3, -2\}$

Step 5: $\frac{2x(x + 3)}{(x + 3)(x + 2)}$ $(x + 3)$ is a binomial factor of the numerator and the denominator. **REDUCE** to 1.

Simplified: $\frac{2x}{x + 2}$

<u>Questions</u>	<u>Answers</u>
1. $\frac{15x}{3x^2}$	$\frac{5}{x}$
2. $\frac{2a - 10}{2}$	$a - 5$
3. $\frac{8k - 16}{k^2 - 4}$	$\frac{8}{k + 2}$
4. $\frac{x^2 - 7x + 12}{2x^2 - 5x - 12}$	$\frac{x - 3}{2x + 3}$

IA Question #3bd: RATIONAL EXPRESSIONS – Multiplication/Division

STEPS FOR MULTIPLYING (or dividing) 2 FRACTIONS:

1. Put parentheses around any fraction bar grouping to remind you it is **ALL** or **NOTHING**.
2. Factor out all **COMMON** factors.
3. Factor out all **BINOMIAL** factors.
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero).
5. If division: Find the reciprocal of the fraction **AFTER** the division sign and put in mult. sign.
6. Multiply your numerators and then multiply your denominators.

NOTE: Do NOT actually multiply groupings, but show to be multiplied.

7. Reduce.

Multiplication example:

$$\frac{x^2 + x}{2x - 8} \cdot \frac{12}{x^2 + 3x + 2}$$

Step 1:
$$\frac{(x^2 + x)}{(2x - 8)} \cdot \frac{12}{(x^2 + 3x + 2)}$$

Step 2&3:
$$\frac{x(x+1)}{2(x-4)} \cdot \frac{12}{(x+2)(x+1)}$$

Step 4: Restrictions: $x \neq \{-2, -1, 4\}$

Step 5:

Step 6:
$$\frac{12x(x+1)}{2(x-4)(x+2)(x+1)}$$

Step 7: The factors **2** and **(x + 1)** are common to the numerator and denominator. **REDUCE** to 1.

Simplified:
$$\frac{6x}{(x-4)(x+2)}$$

Division example:

$$\frac{x^2 + x}{2x - 8} \div \frac{3x}{6x^2 - 12x - 48}$$

$$\frac{(x^2 + x)}{(2x - 8)} \div \frac{3x}{(6x^2 - 12x - 48)}$$

$$\frac{x(x+1)}{2(x-4)} \div \frac{3x}{6(x-4)(x+2)}$$

Restrictions: $x \neq \{-2, 0, 4\}$

$$\frac{x(x+1)}{2(x-4)} \cdot \frac{6(x-4)(x+2)}{3x}$$

$$\frac{6x(x+1)(x-4)(x+2)}{2 \cdot 3x(x-4)}$$

The factors **6**, **x**, and **(x - 4)** are common to the numerator and denominator. **REDUCE** to 1.

$$(x+1)(x+2)$$

STEPS FOR Adding or Subtracting 2 FRACTIONS:

1. Put parentheses around any fraction bar grouping to remind yourself it's **ALL** or **NOTHING**
2. Factor out all **COMMON** factors
3. Factor out all **BINOMIAL** factors
4. Find all restrictions (Values of the variable that will cause the denominator to equal zero)
5. Find a **COMMON** denominator (If you already have one go to step 6)
 - A. Write all factors of **FIRST** denominator
 - B. Multiply by **ANY OTHER** factors of second denominator
 - C. Multiply by **ANY OTHER** factors of subsequent denominators
 - D. Use identity of multiplication to get your fractions to this common denominator
6. Add/Subtract your numerators (Keep your same common denominator)
(Remember that subtraction will **CHANGE** the sign of **EVERY TERM** being subtracted)
7. Simplify **NUMERATOR** (NOT your denominator)
8. Factor numerator
9. Reduce

Example: $\frac{2x-5}{6x+9} - \frac{4}{2x^2+3x} + \frac{1}{x}$

Step 1: $\frac{(2x-5)}{(6x+9)} - \frac{4}{(2x^2+3x)} + \frac{1}{x}$

Step 2&3: $\frac{(2x-5)}{3(2x+3)} - \frac{4}{x(2x+3)} + \frac{1}{x}$

Step 4: Restrictions: $x \neq \{0, -\frac{3}{2}\}$

Step 5A,B,C: Lowest Common Denominator: $3x(2x+3)$

Step 5D: $\frac{(2x-5)}{3(2x+3)} \cdot \frac{x}{x} - \frac{4}{x(2x+3)} \cdot \frac{3}{3} + \frac{1}{x} \cdot \frac{3}{3} \cdot \frac{(2x+3)}{(2x+3)}$

Step 5D: $\frac{x(2x-5)}{3x(2x+3)} - \frac{12}{3x(2x+3)} + \frac{3(2x+3)}{3x(2x+3)}$ (Simplified by multiplication)

Step 6: $\frac{x(2x-5) - 12 + 3(2x+3)}{3x(2x+3)}$

Step 7: $\frac{2x^2 + x - 3}{3x(2x+3)}$

Step 8: $\frac{(x-1)(2x+3)}{3x(2x+3)}$

Step 9: $\frac{x-1}{3x}$

IA Question #3f: Rational Equations

- Find a common denominator using the following steps:
 - Factor out all COMMON and BINOMIAL factors.
 - Write all factors of FIRST denominator.
 - Multiply by **ANY OTHER** factors of subsequent denominators.
- Multiply both sides of the equation (EACH TERM) by this common denominator.
(This step will eliminate all denominators from your equation after you simplify.)
- Solve equation. (Check to see if answer might be a restricted value.)

Example: $\frac{3}{x+2} - \frac{1}{x} = \frac{1}{5x}$

Step 1: Lowest Common Denominator: $(5x)(x+2)$

Step 2: $\frac{5x(x+2)}{1} \cdot \frac{3}{x+2} - \frac{5x(x+2)}{1} \cdot \frac{1}{x} = \frac{5x(x+2)}{1} \cdot \frac{1}{5x}$

Step 2: $5x(3) - 5(x+2) = x+2$ Simplified (Reducing to eliminate denominators)
 $15x - 5x - 10 = x+2$
 $10x - 10 = x+2$

Step 3: $9x - 10 = 2$
 $9x = 12$
 $x = \frac{4}{3}$

*** **Option:** IF YOU HAVE A PROPORTION: FRACTION = FRACTION ***

Example: $\frac{10}{x+4} = \frac{15}{4(x+1)}$

Note: As explained above you COULD multiply BOTH sides of the equation by the LCD.

Optional shortcut for proportion: Cross - Multiply to eliminate denominators.

$$\begin{aligned}4(x+1)(10) &= 15(x+4) \\(4x+4)(10) &= 15x+60 \\40x+40 &= 15x+60 \\25x+40 &= 60 \\25x &= 20 \\x &= \frac{4}{5}\end{aligned}$$

IA Question #4: **FUNCTIONS**

For each input of a function there is one and only one output.

(In other words: Each question has one and only one answer.)

Reading a function:

$f(x)$ is read “ f of x ”

$g(x)$ is read “ g of x ”

$f(m)$ is read “ f of m ”

$g(3)$ is read “ g of 3”

$r(2z - 5)$ is read “ r of $2z - 5$ ”

A specific function is identified by the variable in front of the parentheses.

The input variable is: (inside the parentheses).

What you do with the input variable is determined by the “RULE” that follows the equals sign.

On a graph the function (output) is represented by the vertical or y -axis

Example: $f(x) = x + 4$

Read: f of x equals $x + 4$.

This specific function is called “ f ”.

The input variable is “ x ”.

Rule: $x + 4$.

The output is what $f(x)$ equals.

Whatever value you choose to input for “ x ” will be put into the RULE to find the value (output) of this function.

$$f(6) = 6 + 4$$

$$f(6) = 10$$

$$f(-13) = -13 + 4$$

$$f(-13) = -9$$

$$f(3m - 5) = (3m - 5) + 4$$

$$f(3m - 5) = 3m - 1$$

Example: $g(m) = 2m^2 + 3m - 7$

Read: g of m equals 2 times m squared plus 3 times m minus 7.

This specific function is called “ g ”.

The input variable is “ m ”.

Rule: $2m^2 + 3m - 7$.

The output is what $g(m)$ equals.

Whatever value you choose to input for “ m ” will be put into the RULE to find the value (output) of this function.

$$g(5) = 2(5)^2 + 3(5) - 7$$

$$g(5) = 2(25) + 15 - 7$$

$$g(5) = 50 + 15 - 7$$

$$g(5) = 58$$

$$g(-4z) = 2(-4z)^2 + 3(-4z) - 7$$

$$g(-4z) = 2(16z^2) - 12z - 7$$

$$g(-4z) = 32z^2 - 12z - 7$$

IA Question #5: **Rationalizing an expression**

A rationalized expression means:

No fraction inside the radical.

No radical in the denominator of the fraction.

How to remove a fraction inside the radical:

$$\sqrt{\frac{3}{5}}$$

Example (Not rationalized)

$$\sqrt{\frac{3}{5} \cdot \frac{5}{5}}$$

Multiply numerator and denominator by
the value of the denominator (identity factor).

$$\sqrt{\frac{15}{25}}$$

Multiply

$$\frac{\sqrt{15}}{\sqrt{25}}$$

Separate radicals

$$\frac{\sqrt{15}}{5}$$

Rationalized form

How to remove a radical in the denominator:

$$\frac{3}{\sqrt{5}-4}$$

Example (Not rationalized)

$$\frac{3}{\sqrt{5}-4} \cdot \frac{\sqrt{5}+4}{\sqrt{5}+4}$$

Multiply numerator and denominator by
the conjugate ($\sqrt{5}+4$) of the denominator.

$$\frac{3\sqrt{5}+12}{5-16}$$

Simplify

$$\frac{3\sqrt{5}+12}{11}$$

Rationalized form

IA Question #6: Writing Linear Equations

Parallel lines: Have the same slopes.

Perpendicular lines: Have the opposite and reciprocal slopes.

Slopes of -5 and $\frac{1}{5}$ represent perpendicular lines.

Slopes of $\frac{4}{7}$ and $-\frac{7}{4}$ represent perpendicular lines.

Slope format: $\frac{y_2 - y_1}{x_2 - x_1}$ Use when you know any 2 points.

Equation formats:

Slope-Intercept form: $y = mx + b$ Know slope and y-intercept.

Point-Slope form: $y - y_1 = m(x - x_1)$ Know slope and point (x_1, y_1)

Standard form: $ax + by = c$ No fractional values for a, b, or c and value of 'a' is positive.

Find a linear equation that crosses $(4, -3)_1$ and $(-2, 9)_2$:

$$\text{Slope: } \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-3)}{-2 - 4} = \frac{12}{-6} = -2$$

Since we know the slope = -2 and a point $(4, -3)$, use point-slope form.

$$y - (-3) = -2(x - 4) \quad \text{Substitute slope and point value into form.}$$

$$y + 3 = -2x + 8 \quad \text{Simplified (Equation is not in a specific form).}$$

$$y = -2x + 5 \quad \text{Simplified to Slope-Intercept form.}$$

$$2x + y = 5 \quad \text{Simplified to Standard form.}$$

Find a linear equation that is perpendicular to $y = 3x - 7$ and crosses $(3, 6)$:

Slope of given equation: 3 Slope of a perpendicular line: $-\frac{1}{3}$

Since we know the slope = $-\frac{1}{3}$ and a point $(3, 6)$, use point-slope form.

$$y - 6 = -\frac{1}{3}(x - 3) \quad \text{Substitute slope and point value into form.}$$

$$y - 6 = -\frac{1}{3}x + 1 \quad \text{Simplified (Equation is not in a specific form).}$$

$$y = -\frac{1}{3}x + 7 \quad \text{Simplified to Slope-Intercept form.}$$

$$3(y) = 3(-\frac{1}{3}x + 7) \quad \text{Multiply both sides by 3 to eliminate fraction.}$$

$$3y = -1x + 21 \quad \text{Distributive property}$$

$$x + 3y = 21 \quad \text{Simplified to Standard form.}$$

Setting up Word Problems - Using 2 Variables

1. Draw a picture that matches your information.
2. Put all information from the problem on your picture.
Option: Make a chart for all your information. (Great organizational tool.)
3. Look for relationships among the information. (Definition, formula, logical conclusion)
4. If lost, try assigning **reasonable guesses** to the questions and calculate if they are correct or incorrect. Guessing will give a better understanding of the information.
5. Assign a variable to each of the unknowns and then write two equations that should be similar to the information that you were guessing in the previous step.
6. Solve equations.
7. Answer the original question(s) in sentence form.

Example: Arctic Antifreeze is 18% alcohol and Frost No More is 10% alcohol. How many liters of each should be mixed to get 20 liters of a mixture that is 15% alcohol?

<u>18% Arctic Antifreeze (? Liters)</u>	<u>10% Frost No More (? Liters)</u>	<u>15% Mixture (20 Liters)</u>
Pure alcohol (18% of ? Liters)	Pure alcohol (10% of ? liters)	Pure alcohol (15% of 20 Liters)



Logic: The **percentages** represent the amount of **PURE** alcohol that is mixed with water.

The amount of **pure** alcohol in both original containers should total the amount in the mixture.

Variables: Let **A** be the correct amount of Liters for the *Arctic* and **F** be the correct amount for the *Frost*.

Setup 1: Amount of liters of *Arctic* + Amount of liters of *Frost* = Total liters for mixture

$$A \text{ liters} + F \text{ liters} = 20 \text{ liters}$$

Setup 2: Amount of **pure** alcohol in *Arctic* + Amount of **pure** alcohol in *Frost* = **Pure** alcohol in mixture

$$18\% \text{ of } A \text{ liters} + 10\% \text{ of } F \text{ liters} = 15\% \text{ of } 20 \text{ liters}$$

Example: The Nutty Professor mixes 20 pounds of cashews that cost \$6.75 per pound and some Brazil nuts that cost \$5.15 per pound. If he wants to make a mixture that costs \$5.70 per pound, how many pounds of Brazil nuts should he use?

20 pounds of cashews at \$6.75 ? pounds of Brazil nuts at \$5.15 ? pounds of mixture at \$5.70

Logic: The cost of the cashews plus the cost of the Brazil nuts should equal the total cost of the mixture.



Variables: Let **B** be the correct pounds of Brazil nuts and **M** be the correct pounds for the mixture.

Setup 1: Pounds of cashews + Pounds of Brazil nuts = Pounds of nut mixture

$$20 \text{ pounds} + B \text{ pounds} = M \text{ pounds}$$

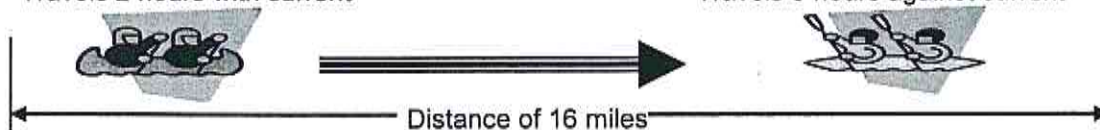
Setup 2: Cost of cashews + Cost of Brazil nuts = Cost of the nut mixture

$$\$6.75 (20 \text{ lbs.}) + \$5.15 (B \text{ lbs.}) = \$5.70 (M \text{ lbs.})$$

Example: A boater travels 16 miles downstream in 2 hours. Returning, against the current, takes 8 hours. What is the speed of the boat in still water and the speed of the river?

Travels 2 hours with current

Travels 8 hours against current



Logic: Distance traveled = Rate of speed times length of Time elapsed. ($D = R \cdot T$)

Downstream travel will add to your boat speed while upstream will subtract from your boat speed.

Variables: Let **B** be the correct speed of the boat in still water and **C** the correct speed of the current.

Setup 1: Distance downstream = Rate downstream · Time downstream

$$16 \text{ miles} = (B + C) \cdot 2 \text{ hours}$$

Setup 2: Distance upstream = Rate upstream · Time upstream

$$16 \text{ miles} = (B - C) \cdot 8 \text{ hours}$$