AP Calculus Sec 2.4 The Chain Rule

The Chain Rule $\frac{d}{dx}\left[f(g(x))\right] = f'\left(g(x)\right)g'(x)$

ex. $y = (3x^2+1)^4$

$y' = 4(3x^2+1)^3(6x)$

$y' = 24x(3x^2+1)^3$

$y = x^4$

$y' = 4x^3$

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General Power Rule

$$\frac{d}{dx}[u^n] = n u^{n-1} u'$$

ex $f(x) = x^2 \cdot \sqrt{9-x^2}$

$f'(x) = x^2\left[\frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)\right] + 2x\sqrt{9-x^2}$

$= \frac{-x^3}{\sqrt{9-x^2}} + \frac{2x\sqrt{9-x^2}\sqrt{9-x^2}}{\sqrt{9-x^2}}$

$= \frac{-x^3 + 2x(9-x^2)}{\sqrt{9-x^2}}$ $\quad 18x - 2x^3$

$= \frac{18x - 3x^3}{\sqrt{9-x^2}}$

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or. $y = \sqrt{\frac{2x}{x+1}} = \left(\frac{2x}{x+1}\right)^{1/2}$

$y' = \frac{1}{2}\left(\frac{2x}{x+1}\right)^{-1/2}\left[\frac{(x+1)(2) - (2x)(1)}{(x+1)^2}\right]$ (2x+2 − 2x)

$y' = \frac{1}{2}\left(\frac{2x}{x+1}\right)^{-1/2}\left[\frac{2 \cdot 1}{(x+1)^2}\right]$

$y' = \frac{1}{(x+1)^2\sqrt{\frac{2x}{x+1}}} = \frac{1}{(x+1)^2\sqrt{2x}\,/\sqrt{x+1}} = \frac{1}{\sqrt{2x(x+1)^3}}$ or $\frac{1}{(x+1)\sqrt{2x(x+1)}}$

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Trigonometric Functions and the Chain Rule

The "Chain Rule versions" of the derivatives of the six trigonometric functions are as follows.

$$\frac{d}{dx}[\sin u] = (\cos u)\,u' \qquad \frac{d}{dx}[\cos u] = -(\sin u)\,u'$$

$$\frac{d}{dx}[\tan u] = (\sec^2 u)\,u' \qquad \frac{d}{dx}[\cot u] = -(\csc^2 u)\,u'$$

$$\frac{d}{dx}[\sec u] = (\sec u \tan u)\,u' \qquad \frac{d}{dx}[\csc u] = -(\csc u \cot u)\,u'$$

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ex. $h(x) = \sec(x^2)$ → note: not the same as $\sec^2 x = \sec x \cdot \sec x$ or $(\sec x)^2$

$h'(x) = \sec x^2 \tan x^2 \cdot 2x$

$h'(x) = 2x \sec x^2 \tan x^2$

$y = x^{1/2}$

$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

ex. $y = \sin\sqrt{x} + \sqrt{\sin x}$

$y = \sin(x^{1/2}) + (\sin x)^{1/2}$

$y' = \cos x^{1/2} \cdot \frac{1}{2\sqrt{x}} + \frac{1}{2}(\sin x)^{-1/2} \cdot \cos x$

$y' = \frac{\cos\sqrt{x}}{2\sqrt{x}} + \frac{\cos x}{2\sqrt{\sin x}}$

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FIND THE DERIVATIVE OF THE GIVEN FUNCTION. EXPRESS YOUR ANSWER IN SIMPLEST FACTORED FORM.

451. $A(z) = (3z - 5)^4$

452. $q(u) = (3u^5 - 2u^3 - 3u - \frac{1}{3})^3$

453. $b(y) = (y^3 - 5)^{-4}$

454. $c(d) = \sqrt[3]{(5d^2 - 1)^5}$

455. $u(p) = \dfrac{3p^2 - 5}{p^3 + 2p - 6}$

456. $V(x) = \dfrac{\sqrt{5x^3}}{5x^3}$

457. $f(x) = 3x^{1/3} - 5x^{-1/3}$

458. $g(z) = \dfrac{1}{\sqrt{36 - z^2}}$

459. $p(t) = (3 - 2t)^{-1/2}$

460. $h(u) = \sqrt{u - 1}\sqrt[3]{2u + 3}$

461. $f(x) = \dfrac{3x}{x + 5}$

462. $g(y) = \dfrac{4y - 3}{3 - 2y}$

463. $p(x) = \dfrac{x^2 + 10x + 25}{x^2 - 10x + 25}$

464. $m(x) = \dfrac{7x}{1 - 3x}$

465. $f(x) = \dfrac{3}{x^2} - \dfrac{x^2}{3}$

466. $g(x) = \left(\dfrac{4x - 3}{5 - 3x}\right)(2x + 7)$

467. $F(x) = 10x^{27} - 25x^{1/5} + 12x^{-12} + 350$

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