# AP Calculus Sec 2.2

## The Constant Rule

$$\frac{d}{dx}[c]=0$$

ex. $\frac{d}{dx}[3]= 0$

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## The Power Rule

$$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^4-x^4}{\Delta x}$

ex. $\frac{d}{dx}\left[x^4\right]= 4x^3$ $\quad \frac{1}{3}-\frac{3}{3}$

ex. $\frac{d}{dx}\left[\sqrt[3]{x}\right]= \frac{d}{dx}\left[x^{\frac{1}{3}}\right] = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$

ex. $\frac{d}{dx}\left[\frac{1}{x^3}\right]= \frac{d}{dx}\left[x^{-3}\right]^{-1} = -3x^{-4} = \frac{-3}{x^4}$

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## The Constant Multiple Rule

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

ex. $\frac{d}{dx}[3x^4] = 3\frac{d}{dx}x^4 = 3(4x^3) = 12x^3$

ex. $\frac{d}{dx}\left[\frac{4x^2}{5}\right] = \frac{4}{5}\left(\frac{d}{dx}x^2\right) = \frac{4}{5}\left(\frac{2x}{1}\right) = \frac{8}{5}x$

ex. $\frac{d}{dx}\left[\frac{1}{2\sqrt[3]{x^2}}\right] = \frac{1}{2}\left[\frac{d}{dx}x^{-2/3}\right]^{-\frac{3}{3}} = \frac{1}{\cancel{2}}\cdot\frac{-\cancel{2}}{3}\left[x^{-5/3}\right] = -\frac{1}{3\sqrt[3]{x^5}}$

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ex. $\frac{d}{dx}\left[\frac{6}{5}\sqrt[4]{x^5}\right]$

$\frac{6}{5}\frac{d}{dx}\left[\sqrt[4]{x^5}\right]$

$\frac{6}{5}\frac{d}{dx}\left[x^{5/4}\right]-\frac{4}{4}$

$^{3}\frac{\cancel{6}}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{4}_2}\left(x^{1/4}\right) = \frac{3}{2}\sqrt[4]{x}$

move reduce

$\frac{d}{dx}\left(\frac{8}{12}\right)\frac{1}{\sqrt[6]{x^{12}}}$

$\frac{2}{3}\frac{d}{dx}\left(x^{-12/6}\right)$

$\frac{2}{3}\frac{d}{dx}\left(x^{-2}\right)$

$\frac{2}{3}\cdot\frac{-2}{1}x^{-3}$

$-\frac{4}{3x^3}$

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## Sum and Difference Rule

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$$

$\frac{d}{dx}\, x^1 = 1x^0 = 1$

ex. $f(x) = x^3 - 4x + 5 \quad find\ f'(x)$

$f'(x) = 3x^2 - 4$

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3-17 odd

ex. $f(x) = 3x^6 - 2x^4 + 7x^2 - x + 9$

$f'(x) = 18x^5 - 8x^3 + 14x - 1$

$f(x) = 3x^2 \perp 3x$

$6x \perp 3$

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## Derivative of Sine and Cosine Functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

ex. $y = x + \cos x, \quad find\ y'$

$y' = 1 - \sin x$

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## Rate of Change

You have seen how the derivative is used to determine slope. The derivative can also be used to determine the rate of change of one variable with respect to another. Applications involving rates of change occur in a wide variety of fields. A few examples are population growth rates, production rates, water flow rates, velocity, and acceleration.

A common use for rate of change is to describe the motion of an object moving in a straight line. In such problems, it is customary to use either a horizontal or a vertical line with a designated origin to represent the line of motion. On such lines, movement to the right (or upward) is considered to be in the positive direction, and movement to the left (or downward) is considered to be in the negative direction.

The function *s* that gives the position (relative to the origin) of an object as a function of time, *t*, is called a position function.

s(t) is called the position function

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velocity is distance/time $V = \frac{d}{t}$

average velocity = $\frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$

ex. If a billiard ball is dropped from a height of 100 feet, its height *s* at time *t* is given by the position function

$$s(t) = -16t^2 + 100$$

where *s* is measure in feet and *t* is measured in seconds. Find the average velocity over each of the following time intervals.

a) given [1, 2] $\frac{[-16(2)^2+100]-[-16(1)^2+100]}{2-1} = \frac{36-84}{2-1} = \frac{-48}{1} = -48$ ft/sec

b) given [1, 1.5] $\frac{[-16(1.5)^2+100]-84}{1.5-1} = \frac{-20}{.5} = -40$ ft sec

c) given [1, 1.1] $\frac{[-16(1.1)^2+100]-84}{1.1-1} = \frac{80.64-84}{0.1} = -33.6$ ft/sec

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velocity or instantaneous velocity $v(t) = s'(t)$

so in other words... to find the velocity take the derivative of the position function.

Position function $s(t) = \frac{1}{2}gt^2 + V_0 + s_0$

speed is the absolute value of velocity, speed cannot be negative.

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**EXAMPLE 10 Using the Derivative to Find Velocity**

At time $t = 0$, a diver jumps from a platform diving board that is 32 feet above the water (see Figure 2.21). The position of the diver is given by

$$s(t) = -16t^2 + 16t + 32 \quad \text{Position function}$$

where $s$ is measured in feet and $t$ is measured in seconds.

**a.** When does the diver hit the water?

**b.** What is the diver's velocity at impact?

a) set $s(t)=0$ to find t.

$-16t^2 + 16t + 32 = 0$

$-16(t^2 - t - 2) = 0$

$-16(t-2)(t+1) = 0$

$t-2=0$ $\quad t+1=0$

$t=2$ $\quad \cancel{t=-1}$

$v(t) = s'(t) = -32t + 16$

$s'(2) = -32(2) + 16$

$= -48$ ft/sec

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