

Sec 2.6 Related Rates

Related rates in a nutshell - taking the derivative of a function with respect to time.

Basics:

Given $y = x^2 - 3x$ find $\frac{dy}{dt}$ when $x = 3$, $\frac{dx}{dt} = 2$

$$\frac{dy}{dt} = 2x \frac{dx}{dt} - 3 \frac{dx}{dt}$$

$$\frac{dy}{dt} = 2(3)(2) - 3(2)$$

$$\frac{dy}{dt} = 6$$

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ex. Given: $y = \sqrt{x}$

Find: (a) $\frac{dy}{dt}$ when $x = 4$ $\frac{dx}{dt} = 3$

$$\frac{dy}{dt} = \frac{1}{2} x^{-1/2} \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{x}} \frac{dx}{dt}$$

(b) $\frac{dx}{dt}$ when $x = 25$ $\frac{dy}{dt} = 2$

$$2 = \frac{1}{2} (25)^{-1/2} \frac{dx}{dt}$$

$$2 = \frac{1}{2} \left(\frac{1}{5}\right) \frac{dx}{dt}$$

$$10 \cdot 2 = \frac{1}{10} \frac{dx}{dt}$$

$$20 = \frac{dx}{dt}$$

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ex. Given: $xy = 4$

$$x \cdot \frac{dy}{dt} + \frac{dx}{dt} \cdot y = 0$$

(a) $\frac{dy}{dt}$ when $x = 8$ $\frac{dx}{dt} = 10$

Need find y $8y = 4$
 $y = \frac{1}{2}$

$$8 \cdot \frac{dy}{dt} + 10 \left(\frac{1}{2}\right) = 0$$

$$8 \frac{dy}{dt} + 5 = 0 \Rightarrow \frac{dy}{dt} = -\frac{5}{8}$$

(b) $\frac{dx}{dt}$ when $x = 1$ $\frac{dy}{dt} = -6$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$$

$$1 \frac{dx}{dt} + 4(-6) = 0$$

$$\frac{dx}{dt} = 24$$

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ex. find $\frac{dy}{dt}$ given: $y = \frac{3x^2 - y}{2x^3}$, when $x = 4, y = 7, \frac{dx}{dt} = 2$

$$\frac{dy}{dt} = \frac{[2x^3] \left[6x \frac{dx}{dt} - 1 \frac{dy}{dt} \right] - [3x^2 - y] \left[6x^2 \frac{dx}{dt} \right]}{[2x^3]^2}$$

$$\frac{dy}{dt} = \frac{[2(4)^3] \left[6(4)(2) - \frac{dy}{dt} \right] - [3(4)^2 - 7] \left[6(4)^2(2) \right]}{[2(4)^3]^2}$$

$$\frac{dy}{dt} = \frac{(128) \left(48 - \frac{dy}{dt} \right) - (41)(192)}{16384}$$

$$\frac{dy}{dt} = \frac{6144 - 128 \frac{dy}{dt} - 7872}{16384}$$

$$16384 \frac{dy}{dt} = -128 \frac{dy}{dt} - 1728$$

$$+128 \frac{dy}{dt} \quad +128 \frac{dy}{dt}$$

$$\frac{16512 \frac{dy}{dt}}{16512} = \frac{-1728}{16512}$$

$$\frac{dy}{dt} = -0.105$$

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Mechanics of related rates - homework

537. If $xy = -3$ and $\frac{dx}{dt} = 1$, then find $\frac{dy}{dt}$ when $x = 6$.

538. If $x^2 - y^2 = 39$ and $\frac{dx}{dt} = 2$, then find $\frac{dy}{dt}$ when $y = 5$.

539. If $\frac{y}{z} = 13$ and $\frac{dz}{dt} = -2$, then find $\frac{dy}{dt}$ when $y = 26$.

2. $y = 4(x^2 - 5x)$ (a) $\frac{dy}{dt}$ when $x = 3$ $\frac{dx}{dt} = 2$

(b) $\frac{dx}{dt}$ when $x = 1$ $\frac{dy}{dt} = 5$

4. $x^2 + y^2 = 25$ (a) $\frac{dy}{dt}$ when $x = 3, y = 4$ $\frac{dx}{dt} = 8$

(b) $\frac{dx}{dt}$ when $x = 4, y = 3$ $\frac{dy}{dt} = -2$

Sep 30-9:09 AM