Lets apply to a word problem:

#16. Volume. The radius *r* of a sphere is increasing at a rate of 3 inches per minute. $V = \frac{4}{3}\pi r^3$ $\frac{dr}{dt} = 3$ in/min

(a) Find the rates of change of the volume when

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

*r* = 9 inches $\frac{dV}{dt} = 4\pi(9)^2(3) = 972\pi$ in$^3$/min

*r* = 36 inches

(b) Explain why the rate of change of the volume of the sphere is not constant even though *dr/dt* is constant.

Oct 7-12:45 PM

**17. *Volume*** A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is (a) 30 centimeters and ~~(b) 60 centimeters~~?

$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 800$ cm$^3$/min $\frac{dr}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

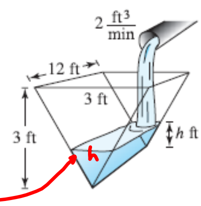
$800 = 4\pi(30)^2 \frac{dr}{dt}$

$\frac{800}{3600\pi} = \frac{\cancel{3600\pi} \frac{dr}{dt}}{\cancel{3600\pi}}$

$\boxed{\frac{2}{9\pi} = \frac{dr}{dt}}$

Oct 15-7:51 AM

#24. Depth. A trough is 12 feet long and 3 feet across the top. Its ends are isosceles triangles with altitudes of 3 feet.




$V = Bh$

$V = \left(\frac{1}{2}bh\right)\cancel{h}\ 12$

$V = \left(\frac{1}{2}h \cdot h\right)12$

$V = 6h^2$

$\frac{dv}{dt} = 12h\frac{dh}{dt}$

3, 3, $h = b$

This is the base

(a) If water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when the depth $h$ is 1 foot?

means volume

$\frac{dv}{dt} = 12h\frac{dh}{dt}$

$2 = 12(1)\frac{dh}{dt}$

$\frac{2}{12} = \frac{dh}{dt} \Rightarrow \boxed{\frac{1}{6}\ \text{ft/min}}$

(b) If the water is rising at a rate of 3/8 inch per minute when h = 2, determine the rate at which water is being pumped into the trough.

Given: $\frac{dh}{dt} = \frac{3}{8}$ in/min need to change to feet to match rest of problem

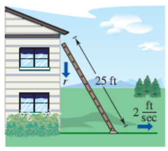
$\hookrightarrow \frac{3}{8} \cdot \frac{1}{12} = \frac{1}{32}$ ft/min

$\frac{dv}{dt} = 12h\frac{dh}{dt}$

$\frac{dv}{dt} = 12(2)\frac{1}{32} = \frac{24}{32} = \boxed{\frac{3}{4}\ \text{ft}^3/\text{min}}$

Oct 7-12:51 PM

#25. Moving ladder. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second.




$\frac{dy}{dt}$, y, 25, x, $\frac{dx}{dt}$

$x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

(a) How fast is the top of the ladder moving down the wall when its base is 7 feet, 15 feet, and 24 feet from the wall?

Find $\frac{dy}{dt}$ Know $\frac{dx}{dt} = 2$ ft/sec

$x = 7$ $y = 24$

$x = 15$ $y = 20$

$x = 24$ $y = 7$

Know Pythagorean Triples

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(7)2 + 2(24)\frac{dy}{dt} = 0$

$28 + 48\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-28}{48} \Rightarrow \boxed{\frac{-7}{12}\ \text{ft/sec}}$

(b) Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.

y 24, 25, x=7

$A = \frac{1}{2}xy$ Given $x = 7, y = 24$ $\frac{dx}{dt} = 2$, $\frac{dy}{dt} = \frac{-7}{12}$

$\frac{dA}{dt} = \frac{1}{2}\left[x\frac{dy}{dt} + \frac{dx}{dt}y\right]$

$\frac{dA}{dt} = \frac{1}{2}\left[7\left(\frac{-7}{12}\right) + 2(24)\right] = \frac{527}{24}\ \text{ft}^2/\text{sec}$

(c) Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall.

y, θ, 25, x

$\tan\theta = \frac{x}{y}$ Given: $x = 7, y = 24, \frac{dx}{dt} = 2, \frac{dy}{dt} = \frac{-7}{24}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{24(2) - 7\left(\frac{-7}{24}\right)}{(24)^2} \cdot \frac{1}{\sec^2\theta}$

$\frac{d\theta}{dt} = \frac{48 + \frac{49}{24}}{24^2}\left(\frac{24}{25}\right)^2 = \boxed{\frac{1}{12}\ \text{rad/sec}}$

Note

$\frac{1}{\sec^2\theta} = \cos^2\theta$

$\cos^2\theta = \left(\frac{24}{25}\right)^2$

Oct 7-12:58 PM