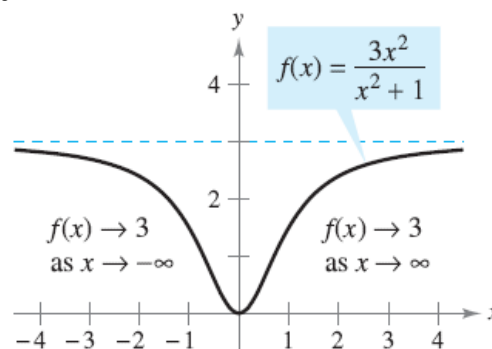


AP Calculus AB Sec 3.5 Limits at Infinity

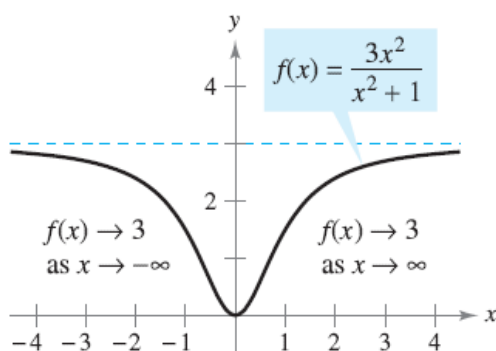
Look at the graph of $f(x) = \frac{3x^2}{x^2 + 1}$



As x goes to $-\infty$, y would approach? 3

As x goes to $+\infty$, y would approach? 3

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So going to $-\infty$ or going to $+\infty$ results in the same answer.

This becomes the definition of a horizontal asymptote.

DEFINITION OF A HORIZONTAL ASYMPTOTE

The line $y = L$ is a **horizontal asymptote** of the graph of f if

$$\lim_{x \rightarrow -\infty} f(x) = L \quad \text{or} \quad \lim_{x \rightarrow \infty} f(x) = L.$$

Talk in your group how this definition would relate to graphing functions. How many horizontal asymptotes could you have?

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Properties of Limits to Infinity:

Basically you can break apart a limit to solve it in a quicker way.

$$\lim_{x \rightarrow \infty} [f(x) + g(x)] = \lim_{x \rightarrow \infty} f(x) + \lim_{x \rightarrow \infty} g(x)$$

$$\lim_{x \rightarrow \infty} [f(x)g(x)] = \left[\lim_{x \rightarrow \infty} f(x) \right] \left[\lim_{x \rightarrow \infty} g(x) \right].$$

THEOREM 3.10 LIMITS AT INFINITY

If r is a positive rational number and c is any real number, then

$$\lim_{x \rightarrow \infty} \frac{c}{x^r} = 0.$$

Furthermore, if x^r is defined when $x < 0$, then

$$\lim_{x \rightarrow -\infty} \frac{c}{x^r} = 0.$$

These are really important make sure you memorize.

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example: $\lim_{x \rightarrow \infty} \left(5 - \frac{2}{x^2} \right) = \lim_{x \rightarrow \infty} 5 - \lim_{x \rightarrow \infty} \frac{2}{x^2}$
 $5 - 0 = 5$

example: $\lim_{x \rightarrow \infty} \frac{2x-1}{x+1} = \frac{\infty}{\infty}$ indeterminate form

Choice 1

$$x+1 \overline{) 2x-1} \\ \underline{-2x+2} \\ -3$$

$$= 2 + \frac{-3}{x+1}$$

$$\lim_{x \rightarrow \infty} 2 + \lim_{x \rightarrow \infty} \frac{-3}{x+1}$$

$$2 + 0 = 2$$

Choice 2

$$\lim_{x \rightarrow \infty} \frac{\frac{2x-1}{x}}{\frac{x+1}{x}} = \lim_{x \rightarrow \infty} \frac{2 - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{2}{1} = 2$$

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Now in your groups try these 3:

$$\lim_{x \rightarrow \infty} \frac{2x+5}{3x^2+1} = \lim_{x \rightarrow \infty} \frac{\frac{2x+5}{x^2}}{\frac{3x^2+1}{x^2}} = \frac{\frac{2}{x} + \frac{5}{x^2}}{3 + \frac{1}{x^2}} = \frac{0+0}{3+0} = 0$$

So Mathematicians saw a pattern with the answers and something else.

$$\lim_{x \rightarrow \infty} \frac{2x^2+5}{3x^2+1} = \lim_{x \rightarrow \infty} \frac{\frac{2x^2+5}{x^2}}{\frac{3x^2+1}{x^2}} = \frac{2 + \frac{5}{x^2}}{3 + \frac{1}{x^2}} = \frac{2+0}{3+0} = \frac{2}{3}$$

Discuss in your groups what that might be.

$$\lim_{x \rightarrow \infty} \frac{2x^3+5}{3x^2+1} = \lim_{x \rightarrow \infty} \frac{\frac{2x^3+5}{x^2}}{\frac{3x^2+1}{x^2}} = \frac{2x + \frac{5}{x^2}}{3 + \frac{1}{x^2}} = \frac{\infty+0}{3+0} = \frac{\infty}{3} = \text{DNE}$$

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Lets review the horizontal asymptote rules from Pre-Calculus:

limits are the same

$$y = \frac{ax^m + \dots}{bx^n + \dots} \quad \text{if} \quad \begin{cases} m < n, & \text{then } y = 0 \\ m = n, & \text{then } y = \frac{a}{b} \\ m > n, & \text{then no horizontal asy } \text{DNE} \end{cases}$$

So Bottom line... the limit to infinity for a rational function is the same as finding the horizontal asymptote.

Unfortunately this only works with rational functions.

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What about Irrational functions?

example: $\lim_{x \rightarrow \infty} \frac{3x-2}{\sqrt{2x^2+1}}$

note: if $x > 0$ then $\sqrt{x^2} = x$

$$\lim_{x \rightarrow \infty} \frac{\frac{3x-2}{x}}{\frac{\sqrt{2x^2+1}}{\sqrt{x^2}}} = \frac{\frac{3x}{x} - \frac{2}{x}}{\sqrt{\frac{2x^2}{x^2} + \frac{1}{x^2}}} = \lim_{x \rightarrow \infty} \frac{3 - \frac{2}{x}}{\sqrt{2 + \frac{1}{x}}} = \frac{3-0}{\sqrt{2+0}} = \frac{3}{\sqrt{2}}$$

example: $\lim_{x \rightarrow -\infty} \frac{3x-2}{\sqrt{2x^2+1}}$

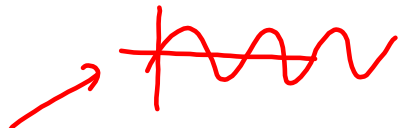
note: if $x < 0$ then $\sqrt{x^2} = -x$

$$\lim_{x \rightarrow -\infty} \frac{\frac{3x-2}{-x}}{\frac{\sqrt{2x^2+1}}{\sqrt{x^2}}} = \frac{\frac{3x}{-x} + \frac{2}{x}}{\sqrt{\frac{2x^2}{x^2} + \frac{1}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{-3 + \frac{2}{x}}{\sqrt{2 + \frac{1}{x}}} = \frac{-3+0}{\sqrt{2+0}} = \frac{-3}{\sqrt{2}}$$

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Limits involving Trig Functions

examples:



1. $\lim_{x \rightarrow \infty} \sin x = \text{DNE}$ because graph oscillates between 1 and -1

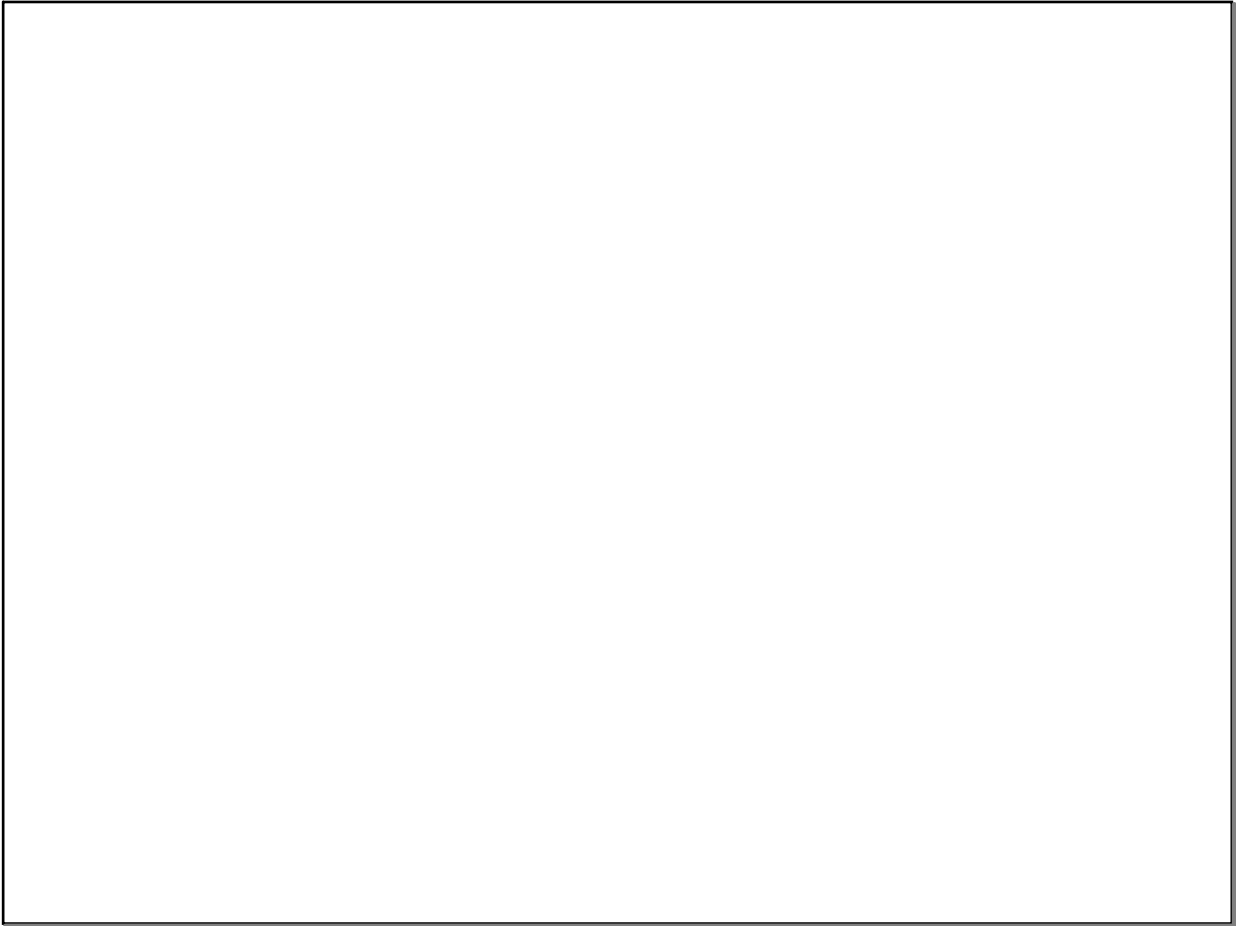
2. $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$ because we know $-\frac{1}{x} < \frac{\sin x}{x} < \frac{1}{x}$
 Squeeze theorem

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\lim_{x \rightarrow \infty} -\frac{1}{x} = 0$$

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