Applications of Derivatives Worksheet Name ______________________________

I. Velocity and Acceleration

a. To find the velocity, take the derivative of the position function

i. Instantaneous velocity is the velocity (derivative of position function) at a moment in time, usually given.

ii. Average Rate of Change (sometimes confused with velocity). $= \frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$ This is not taking the derivative to solve.

b. To find the acceleration, take the derivative of the velocity.

c. Position function if not given is in the form of: $f(x) = -16t^2 + V_o t + S_o$ where V_o, is the initial velocity and S_o is the initial position (height).

Example: At time t = 0, a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$ where s is measured in feet and t is measured in seconds.

a. When does the diver hit the water?
b. What is the diver's velocity at impact?

Solution

a. To find the time t when the diver hits the water, let s = 0 and solve for t.

$-16t^2 + 16t + 32 = 0$
$-16(t^t – t – 2) = 0$
$-16(t – 2)(t + 1) = 0$ so t = 2 or t = -1. Since can't go back in time, t = -1 won't work, so t = 2

b. The velocity at time t is given by the derivative:

$v(t) = -32t + 16$, then substitute t = 2 to find the velocity at impact.
$v(2) = -32(2) + 16 =$ -48 ft/sec

Do the following problems, note that the position function $s(t) = -4.9t^2 + v_o + s_o$ for free-falling objects.

1. A projectile is shot upward from the surface of Earth with an initial velocity of 120 meters per second. What is the velocity after 5 seconds? After 10 seconds?

2. To estimate the height of a building, a stone is dropped from the top of the building into a pool of water at ground level. How high is the building if the splash is seen 5.6 seconds after the stone is dropped?

II. Related Rates – These problems are using implicit differentiation as well as all other rules of taking the derivative. Here are some guidelines:

GUIDELINES FOR SOLVING RELATED-RATE PROBLEMS

1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time t*.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

Example: All edges of a cube are expanding at a rate of 6 cm/sec. How fast is the volume changing when each edge is (a) 2 cm and (b) 10 cm

First what are we looking for and what formula is needed: volume of a cube: V = l x w x h = s^3

We are looking for $\frac{dV}{dt}$. We are given: $\frac{ds}{dt} = 6 \; and \; that \; s = 2, then \; 10$.

$V = s^3$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

a) When s = 2 $\frac{dV}{dt} = 3(2)^2(6) = 3(4)(6) = 72 \; cm/sec^3$

b) When s = 10 $\frac{dV}{dt} = 3(10)^2(6) = 3(100)(6) = 1800 \; cm/sec^3$

Here are some basic volume formulas:

Figure	Picture	Surface Area	Volume
Rectangular Prism		$2lw + 2wh + 2lh$	Area of Base x h V = lwh
Triangular Prism		$bh + (P base)h$ Area of 2 triangles x 2 perimeter	Area of Base x prism height 1/2 bh x h
Cylinder		$2\pi r^2 + 2\pi rh$ Area each circle + circumference x height	V= $\pi r^2 h$ Area of Base x height
Cone		$\pi rs + \pi r^2$ s is the slant height	$\frac{1}{3}\pi r^2 h$
Sphere		$4\pi r^2$	$\frac{4}{3}\pi r^3$

See attached sheet for additional Related Rates practice problems.