AP Calculus Sec 1.5 Infinite Limits

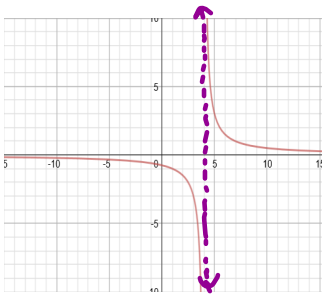
Infinite limit - a limit that increases or decreases without bound.

note: lim f(x) = ∞ does not mean that the limit exists, but how it fails to exist.

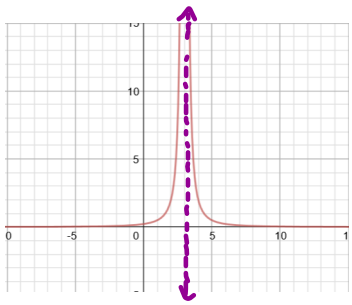
Lets look at the graph of these functions:

lim $x \to 4^+$

$f(x) = \frac{3}{x-4}$ VA $x \neq 4$



$f(x) = \frac{2}{(x-3)^2}$ VA $x \neq 3$



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Vertical Asymptotes

If the function is rational (fraction), there will probably be at least one asymptote.

Reminder:
To find a vertical asymptote, set the denominator equal to zero and solve for x.

example: find the vertical asymptote(s):

$f(x) = \frac{1}{2(x+1)}$

$2(x+1) = 0$

$x = -1$

~~2≠0~~ $x+1=0$

$f(x) = \frac{x^2+1}{x^2-1} = 0$

$x^2 = 1$

$x = \pm 1$

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Before finding the vertical asymptotes should always try to reduce the function.

If when reducing, the denominator cancels with a factor of the numerator, is it still an asymptote?

ex. $f(x)=\frac{x^2+2x-8}{x^2-4}$ $=\frac{(x+4)(x-2)}{(x+2)(x-2)}$

asymptote if it doesn't cancel

hole if it does cancel

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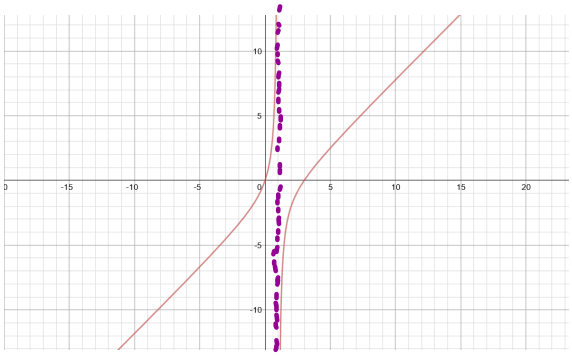
Applying your knowledge of vertical asymptotes find the limits:

$\lim_{x\to 1^-}\frac{x^2-3x}{x-1}$ Answer ∞ or $-\infty$ and $\lim_{x\to 1^+}\frac{x^2-3x}{x-1}$ $= -\infty$

VA = x = 1

If asked $\lim_{x\to 1}\frac{x^2-3x}{x-1}$ = DNE because the limit is different from each side.



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If $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = L$

Rules: $\lim_{x\to c}\left[f(x) \pm g(x)\right] = \infty$ where $L > 0$

$\infty \pm L$

$\lim_{x\to c}\left[f(x)\,g(x)\right] = \infty$ where $L > 0$

$\infty \cdot L$

$\lim_{x\to c}\left[\dfrac{g(x)}{f(x)}\right] = 0$

$\frac{L}{\infty}$ ← the larger the #, the smaller the fraction

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ex. $\lim_{x\to 0^-}\left(x^2 - \dfrac{1}{x}\right)$

$\lim_{x\to 0^-} x^2 - \lim_{x\to 0^-} \frac{1}{x}$

$0 - -\infty = \infty$

$\frac{1}{\frac{1}{10}} = 1 \cdot \frac{10}{1} = \infty$

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