

## Optimization Problems Practice

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**Solve each optimization problem.**

- 1) A company has started selling a new type of smartphone at the price of  $\$110 - 0.05x$  where  $x$  is the number of smartphones manufactured per day. The parts for each smartphone cost \$50 and the labor and overhead for running the plant cost \$6000 per day. How many smartphones should the company manufacture and sell per day to maximize profit? (Remember that Profit = Revenue - Cost)
  
  
  
  
  
  
  
  
  
  
- 2) A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?
  
  
  
  
  
  
  
  
  
  
- 3) A cryptography expert is deciphering a computer code. To do this, the expert needs to minimize the product of a positive rational number and a negative rational number, given that the positive number is exactly 8 greater than the negative number. What final product is the expert looking for?

- 4) A rancher wants to construct two identical rectangular corrals using 400 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?
- 5) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold  $500 \text{ ft}^3$  of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?
- 6) Which point on the graph of  $y = \sqrt{x}$  is closest to the point  $(5, 0)$ ?

- 7) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 8. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?
- 8) Two vertical poles, one 4 ft high and the other 16 ft high, stand 15 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?
- 9) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 18 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

## Answers to Optimization Problems Practice

- 1)  $p$  = the profit per day     $x$  = the number of items manufactured per day  
 Function to maximize:  $p = x(110 - 0.05x) - (50x + 6000)$  where  $0 \leq x < \infty$   
 Optimal number of smartphones to manufacture per day: 600
- 2)  $A$  = the total area of the two corrals     $x$  = the length of the non-adjacent sides of each corral  
 Function to maximize:  $A = 2x \cdot \frac{200 - 4x}{3}$  where  $0 < x < 50$   
 Dimensions of each corral: 25 ft (non-adjacent sides) by  $\frac{100}{3}$  ft (adjacent sides)
- 3)  $P$  = the product of the two numbers     $x$  = the positive number  
 Function to minimize:  $P = x(x - 8)$  where  $-\infty < x < \infty$   
 Smallest product of the two numbers: -16
- 4)  $A$  = the total area of the two corrals     $x$  = the length of the non-adjacent sides of each corral  
 Function to maximize:  $A = 2x \cdot \frac{400 - 4x}{3}$  where  $0 < x < 100$   
 Dimensions of each corral: 50 ft (non-adjacent sides) by  $\frac{200}{3}$  ft (adjacent sides)
- 5)  $A$  = the area of the glass     $x$  = the length of the sides of the square bottom  
 Function to minimize:  $A = x^2 + 4x \cdot \frac{500}{x^2}$  where  $0 < x < \infty$   
 Dimensions of the aquarium: 10 ft by 10 ft by 5 ft tall
- 6)  $d$  = the distance from point  $(5, 0)$  to a point on the curve     $x$  = the  $x$ -coordinate of a point on the curve  
 Function to minimize:  $d = \sqrt{(x - 5)^2 + (\sqrt{x})^2}$  where  $-\infty < x < \infty$   
 Point on the curve that is closest to the point  $(5, 0)$ :  $\left(\frac{9}{2}, \frac{3\sqrt{2}}{2}\right)$
- 7)  $A$  = the area of the rectangle     $x$  = half the base of the rectangle  
 Function to maximize:  $A = 2x\sqrt{8^2 - x^2}$  where  $0 < x < 8$   
 Area of largest rectangle: 64
- 8)  $L$  = the total length of rope     $x$  = the horizontal distance from the short pole to the stake  
 Function to minimize:  $L = \sqrt{x^2 + 4^2} + \sqrt{(15 - x)^2 + 16^2}$  where  $0 \leq x \leq 15$   
 Stake should be placed: 3 ft from the short pole (or 12 ft from the long pole)
- 9)  $A$  = the area of the composite window     $x$  = the width of the bottom window = the diameter of the top window  
 Function to maximize:  $A = x\left(\frac{18}{2} - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\pi \cdot \left(\frac{x}{2}\right)^2$  where  $0 < x < \frac{72}{4 + \pi}$   
 Dimensions of the bottom window:  $\frac{36}{4 + \pi}$  ft (width) by  $\frac{18}{4 + \pi}$  ft (height)