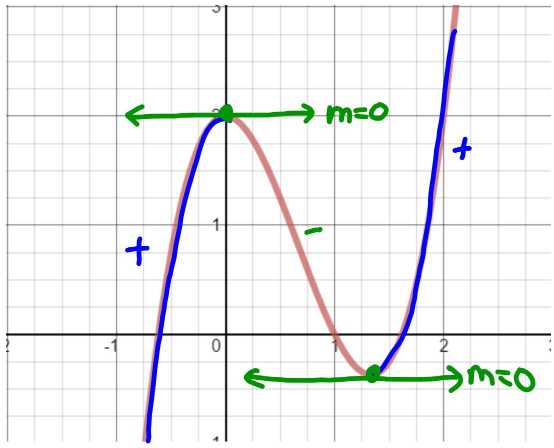
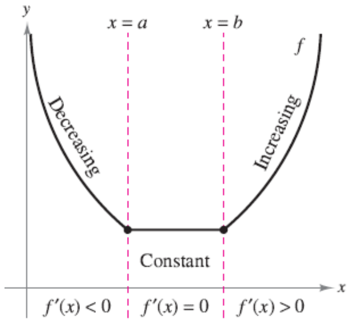
AP Calculus BC Sec 3.3 Increasing and Decreasing Functions and the First Derivative Test



What happens at a change between increase/decrease?

Another reason to love derivatives... Taking the first derivative, this can be used as a test for where the graph is increasing and/or decreasing.



THEOREM 3.5 TEST FOR INCREASING AND DECREASING FUNCTIONS

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

Lets try an example:

Determine when the function is increasing/decreasing

$f(x) = (x - 3)^3$

$f'(x) = 3(x-3)^2(1) = 0$

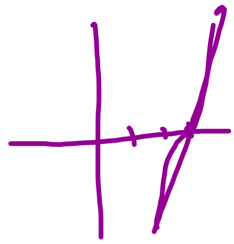
$(x-3)^2 = 0$

$x - 3 = 0$

C.P. $x = 3$

Increasing + ↗ + ↗ increasing

$f'(x) =$ 0 3 4



* When graph is always + or -, then this is called strictly monotonic.

example: Determine when the function is increasing/decreasing

$f(x) = x^4 - 32x + 4$

$f'(x) = 4x^3 - 32 = 0$

$4x^3 = 32$

$x^3 = 8$

C.P. $x = 2$

↘ − + ↗

$f'(x) =$ 0 2 3

f(x) is decreasing $(-\infty, 2)$

f(x) is increasing $(2, \infty)$

Do the following problems as a group on your white boards:

Find the interval of increasing/decreasing on the function

1. $f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

$3x^2 - 3x = 0$

$3x(x-1) = 0$

$3x = 0 \quad x - 1 = 0$

$x = 0 \quad x = 1$

$f'(-1) \quad f'(\tfrac{1}{2}) \quad f'(2)$

$+ \quad 0 \quad - \quad 1 \quad +$

increase: $(-\infty, 0) \cup (1, \infty)$

decrease: $(0, 1)$

Thus the

1st Derivative Test

was born...

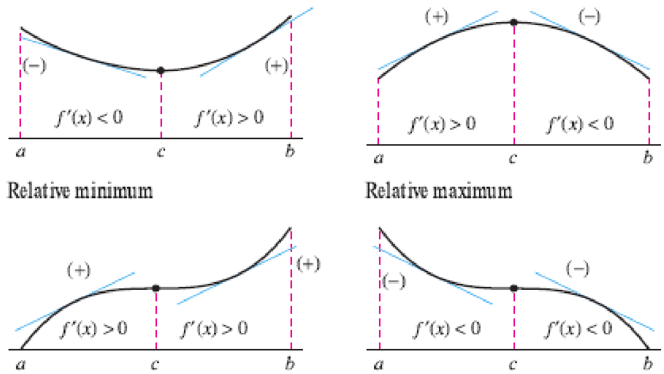
This tells you if a

critical point is a max or min point.

THEOREM 3.6 THE FIRST DERIVATIVE TEST

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then f has a *relative minimum* at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a *relative maximum* at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.



Relative minimum Relative maximum

Neither relative minimum nor relative maximum examples:

Find all relative max/min points for:

$f(x) = 2x^3 - 3x^2 - 36x + 14$

$f'(x) = 6x^2 - 6x - 36$

$6(x^2 - x - 6) = 0$

$(x+2)(x-3) = 0$

$x = -2 \quad x = 3$

$+ \quad - \quad +$

x: $-3 \quad -2 \quad 0 \quad 3 \quad 4$

$-2 = \text{max} \quad 3 = \text{min}$

$(-2, 13) \quad (3, -67)$

example:

Find all relative extrema for: $f(x) = (x^2 - 4)^{\frac{2}{3}}$

$f'(x) = \frac{2}{3}(x^2-4)^{-1/3}(2x)$

$\frac{4x}{3\sqrt[3]{x^2-4}} = \frac{0}{1}$

$4x(1) = 0(3\sqrt[3]{x^2-4})$

$4x = 0$

C.P. $x = 0$

$f'(x)$: $- \quad +$

$x =$ $-1 \quad 0 \quad 1$

Therefore $x = 0$ is a minimum

$(0, 2.52)$

Do in your group on the white boards the following problems:

Find all relative extrema:

1. $f(x) = x^2 + 6x + 10$

$f'(x) = 2x + 6 = 0$

$x = -3$

$f'(x)$ − +

$x = -4 \quad -3 \quad 0$

$x = -3$ is a min

2. $f(x) = x^4 - 32x + 4$

$f'(x) = 4x^3 - 32 = 0$

$x^3 = 8$

$x = 2$

$f'(x)$ − +

$x = 0 \quad 2 \quad 3$

$x = 2$ is a min

assignment: page 210 (online) page 186 (book) 3, 5, 17-29 odd, 35, 37

AP Calculus Problem book page 70

3.3 The First and Second Derivative Tests

FOR THE FOLLOWING, FIND: ~~A) THE DOMAIN OF EACH FUNCTION;~~ B) THE x-COORDINATE OF THE LOCAL EXTREMA, AND C) THE INTERVALS WHERE THE FUNCTION IS INCREASING AND/OR DECREASING.

701. $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x - 1$

702. $g(x) = x^3 - 5x^2 - 8x$

703. $h(x) = x + \dfrac{4}{x}$

704. $p(x) = \sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}}$

705. $h(x) = (2 - x)^2(x + 3)^3$

706. $m(x) = 3x\sqrt{5 - x}$

707. $f(x) = x^{2/3}(x - 5)^{-1/3}$

708. $h(x) = \frac{1}{7}x^{7/3} - x^{4/3}$