**THEOREM 3.4 THE MEAN VALUE THEOREM**

If $f$ is continuous on the closed interval $[a, b]$ and differentiable on the open interval $(a, b)$, then there exists a number $c$ in $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

1. $f$ is continuous on $[a, b]$
2. $f$ is differentiable on $(a, b)$

then $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

example: Apply the mean value theorem, find c.

$f(x) = x(x^2 - x - 2)$ on the interval $[\overset{a}{-1},\overset{b}{1}]$

$f(x) = x^3 - x^2 - 2x$

1. Because $f(x)$ is a polynomial it is both continuous and differentiable on $[-1, 1]$

$f'(x) = \dfrac{f(b) - f(a)}{b - a}$

$3x^2 - 2x - 2 = \dfrac{[1^3 - (1)^2 - 2(1)] - [(-1)^3 - (-1)^2 - 2(-1)]}{1 - (-1)}$

$3x^2 - 2x - 2 = \dfrac{-2 - 0}{2}$

$3x^2 - 2x - 2 = -1$
$\quad\quad\quad\quad\;\; +1$

$3x^2 - 2x - 1 = 0$ $\quad -3$: $-3,\ 1$

$\left(\dfrac{3x - 3}{3}\right)(3x + 1) = 0$

$(x - 1)(3x + 1) = 0$

$x = 1, -\frac{1}{3}$

You Try: Apply the mean value theorem, find c.

$f(x) = x^3$ on $[0,1]$

① $\frac{1^3 - 0^3}{1 - 0} = \frac{1}{1} = 1$ $\frac{f(b)-f(a)}{b-a}$

② $f'(x) = 3x^2$

③ $3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} \rightarrow \frac{\sqrt{3}}{3}$

In the following eight problems, verify the two conditions required by the Mean Value Theorem and then find a suitable number $c$ guaranteed to exist by the Mean Value Theorem.

**687.** $f(x) = 4x^2 - x - 6$ on $[1,3]$

**688.** $g(x) = \dfrac{x-1}{x+2}$ on $[0,2]$

**689.** $p(x) = 3x^{2/3} - 2x$ on $[0,1]$

**690.** $k(x) = x^4 - 3x$ on $[1,3]$

**691.** $F(x) = x^3$ on $[1,3]$

**692.** $G(x) = (x-1)^3$ on $[-1,2]$

**693.** $P(x) = x^2 + 5x$ on $[0,2]$

**694.** $H(x) = x^3$ on $[-1,3]$

Find critical points of the functions in the following four problems.

**695.** $f(x) = 3x^2 - 5x + 1$

**696.** $h(x) = x^4 - 2x^2 + 3$

**697.** $p(x) = \dfrac{3x-2}{x-4}$

**698.** $h(x) = 2x^{5/3} - x^{2/3} + 3$

**700.** A trucker handed in a ticket at a toll booth showing that in 2 hours he had covered 159 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?