AP Calculus BC

Sec 3.2 Rolle's Theorem and the Mean Value Theorem

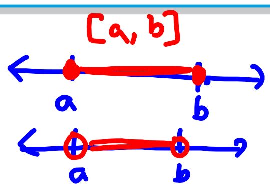
THEOREM 3.3 ROLLE'S THEOREM

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If

$$f(a) = f(b)$$

then there is at least one number c in (a, b) such that $f'(c) = 0$.

If
1. continuous on $[a, b]$
2. differentiable on (a, b)
3. $f(a) = f(b)$



then $f'(c) = 0$

example: If Rolle's Theorem can be applied, find all values of c in the interval such that f'(c) = 0,

$f(x) = x^2 - 3x + 2$ on [1, 2]

① $f(x)$ is continuous on $[1, 2]$

② $f(x)$ is differentiable on $(1, 2)$

③ $f(1) = f(2)$

$(1)^2 - 3(1) + 2 = (2)^2 - 3(2) + 2$

$0 = 0$ ✓

④ $f'(x) = 2x - 3 = 0$

$x = \frac{3}{2}$

example: If Rolle's Theorem can be applied, find all values of *c* in the interval such that f'(*c*) = 0

$f(x) = \sec x$ on $[-\pi/4, \pi/4]$

① f(x) is continuous on $[-\frac{\pi}{4}, \frac{\pi}{4}]$

② f(x) is differentiable on $(-\frac{\pi}{4}, \frac{\pi}{4})$

③ $f(a) = f(b)$

$\sec(-\frac{\pi}{4}) = \sec(\frac{\pi}{4})$ because even function they are =

④ $f'(x) = \sec x \tan x = 0$

$\cancel{\sec x = 0}$ $\tan x = 0$

$x = 0, \cancel{\pi}$ $\Rightarrow c = 0$

You try:

Determine whether Rolle's Theorem can be applied to f on the closed interval [a,b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a,b) such that f'(c) = 0.

$f(x) = x^2 - 5x + 4$ [1,4]

① f(x) is continuous on [1,4]

② f(x) is differentiable on (1,4)

③ $f(1) = f(4)$

$1^2 - 5(1) + 4 = 4^2 - 5(4) + 4$

$0 = 0$ ✓

④ $f'(x) = 2x - 5 = 0$

$x = \frac{5}{2}$ and is between 1 and 4 therefore $\boxed{c = 5/2}$

3.2 Rolle to the Extreme with the Mean Value Theorem

In the following four problems, verify the three conditions required by Rolle's Theorem and then find a suitable number c guaranteed to exist by Rolle's Theorem.

683. $f(x) = 2x^2 - 11x + 15$ on $[\frac{5}{2}, 3]$

684. $g(x) = x^3 + 5x^2 - x - 5$ on $[-5, -1]$

685. $p(x) = 4x^{4/3} - 6x^{1/3}$ on $[0, 6]$

686. $k(x) = \dfrac{x^2 - 4}{x^2 + 4}$ on $[-2, 2]$