Sec 6.2 Differential Equations: Growth and Decay

Solving a Differential Equation means $y = \int y'$

$$\boxed{y' = \frac{dy}{dx}}$$

$$y' = \frac{2x}{y}$$

$$\frac{dy}{dx} = \frac{2x}{y}$$

$$\int y\,dy = \int 2x\,dx$$

$$\frac{y^2}{2} = \frac{\cancel{2}x^2}{\cancel{2}} + C$$

$$2\left(\frac{1}{2}y^2 - x^2 = C\right)$$

$$\boxed{y^2 - 2x^2 = C}$$

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1, 3, 5, 7

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Exponential Growth and Decay Model

$$y = Ce^{kt}$$

C- initial value

K- proportionality constant

with respect to K

K > 0 growth

K < 0 decay

t - time

ex. The rate of change of y is proportional to y.

When $\boxed{t = 0, y = 2}$ and when $\boxed{t = 2, y = 4}$.

What is the value of $\boxed{y \text{ when } t = 3}$?

$$y = Ce^{kt}$$

$$2 = C\cancel{e^{k(0)}}^{1}$$

$$\boxed{2 = C}$$

$$y = 2e^{kt}$$

$$\frac{4}{2} = \frac{\cancel{2}e^{k(2)}}{\cancel{2}}$$

$$\ln 2 = \cancel{\ln e}^{2k}$$

$$\frac{\ln 2}{2} = \frac{2k}{2}$$

$$\boxed{\tfrac{1}{2}\ln 2 = k}$$

$$y = 2e^{\frac{1}{2}\ln 2(3)}$$

$$\boxed{y = 2e^{\frac{3}{2}\ln 2}} \text{ or } \boxed{5.657}$$

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3. $\frac{dy}{dx} = \frac{y+3}{1}$

$$\frac{dy}{y+3} = dx$$

$$\int \frac{1}{y+3}\,dy = \int dx$$

$$\cancel{e^{\ln}}|y+3| = e^{(x+C)}$$

$$y+3 = e^{x+c}$$

$$y = e^{x+c} - 3$$

$$y = e^x \cdot e^c - 3$$

$$y = e^x C - 3$$

$$\boxed{y = Ce^x - 3}$$

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7. $y' = \sqrt{x}\, y$

$$\frac{dy}{dx} = \frac{\sqrt{x}\, y}{1}$$

$$\frac{dy}{y} = \sqrt{x}\, dx$$

$$\int \frac{1}{y}\,dy = \int x^{1/2}\,dx$$

$$\cancel{e^{\ln}}|y| = e^{\left(\frac{2}{3}x^{3/2} + C\right)}$$

$$y = e^{\left(2/3\, x^{3/2} + c\right)}$$

$$y = Ce^{2/3\, x^{3/2}} \quad \text{or} \quad Ce^{\frac{2x^{3/2}}{3}}$$

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21. When $x=0,\ y=6$ when $x=4,\ y=15$

What is the value of y when $x=8$

$y = Ce^{kx}$

$6 = C\cancel{e^{k(0)}}$

$\boxed{C=6}$

$y = 6e^{kx}$

$\frac{15}{6} = \frac{\cancel{6}e^{k(4)}}{\cancel{6}}$

$\ln\frac{5}{2} = \cancel{\ln e}^{4k}$

$\frac{\ln\frac{5}{2}}{4} = \frac{\cancel{4}k}{\cancel{4}}$

$\frac{\ln\frac{5}{2}}{4} = k$

$y = 6e^{\frac{\ln\frac{5}{2}}{\cancel{4}}(\cancel{8})^{2}}$

$y = 6e^{2\ln\frac{5}{2}}$

$y = 6\cancel{e^{\ln}}\left(\frac{5}{2}\right)^2$

$y = 6\left(\frac{5}{2}\right)^2 = \cancel{6}^{3}\left(\frac{25}{\cancel{4}_{2}}\right) = \frac{75}{2}$

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EXAMPLE 3 Radioactive Decay

Suppose that 10 grams of the plutonium isotope ^{239}Pu was released in the Chernobyl nuclear accident. How long will it take for the 10 grams to decay to 1 gram?

$y = \cancel{C}^{10}e^{kt}$

$5 = 10e^{k(24,100)}$

$\ln\frac{1}{2} = \cancel{\ln e}^{24,100k}$

$\frac{\ln\frac{1}{2}}{24,100} = k$

$k \approx -0.000028761$

$t = 0 \quad y = 10$ same as $C=10$

$t = ? \quad y = 1$

$t = 24,100 \quad y = 5$

$1 = 10e^{-0.000028761t}$

$\frac{1}{10} = e^{-0.000028761t}$

$\ln\frac{1}{10} = -0.000028761t$

$t = 80,059$ years

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EXAMPLE 4 Population Growth

Suppose an experimental population of fruit flies increases according to the law of exponential growth. There were 100 flies after the second day of the experiment and 300 flies after the fourth day. Approximately how many flies were in the original population?

$y = Ce^{kt}$

$t = 2 \quad y = 100$

$t = 4 \quad y = 300$

$t = 0 \quad y = ?$

$\frac{100}{e^{2k}} = \frac{Ce^{2k}}{\cancel{e^{2k}}}$

$100e^{-2k} = C$

$300 = Ce^{4k}$

$300 = 100e^{-2k}e^{4k}$

$\frac{300}{100} = \frac{\cancel{100}e^{2k}}{\cancel{100}}$

$\ln 3 = \ln e^{2k}$

$\ln 3 = 2k$

$\frac{\ln 3}{2} = k$

$100 = Ce^{\frac{1}{2}\ln 3(2)}$

$100 = C\cancel{e^{\ln 3}}$

$\frac{100}{3} = \frac{3C}{3}$

$C = \frac{100}{3} = 33.3$

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