Integration By Parts

$$\int u\, dv = uv - \int v\, du$$

u differentiates to zero (usually).

dv is easy to integrate.

The Integration by Parts formula is a "product rule" for integration.

Choose u in this order: LIPET

Logs, **I**nverse trig, **P**olynomial, **E**xponential, **T**rig

ex. $\int xe^x dx$

$u = x \Rightarrow du = 1\, dx$

$dv = e^x dx \Rightarrow \int e^x \Rightarrow v = e^x$

now put in formula:

$uv - \int v\, du$

$x \cdot e^x - \int e^x \cdot 1\, dx$

$\boxed{xe^x - e^x + C}$

ex. $\int x^2 \ln x \, dx$

$u = \ln x \quad du = \frac{1}{x}dx$

$dv = x^2 dx \quad v = \int x^2 dx = \frac{1}{3}x^3$

Put in formula:

$uv - \int v\,du$

$\ln x\left(\frac{1}{3}x^3\right) - \int \frac{1}{3}x^3 \frac{1}{x}dx$

$\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\,dx$

$\frac{1}{3}x^3 \ln x - \frac{1}{3}\left(\frac{1}{3}x^3\right) + C$

$\boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$

Repeated application of integration by parts

ex. $\int x^2 \sin x \, dx$

$u = x^2 \Rightarrow du = 2x\,dx$

$dv = \sin x\,dx \Rightarrow \int \sin x\,dx = -\cos x$

$uv - \int v\,du$

$x^2(-\cos x) - \int -\cos x \; 2x\,dx$

$-x^2\cos x + 2\int \cos x \cdot x\,dx$

$u = x \quad du = dx$

$dv = \cos x \quad v = \int \cos x = \sin x$

$-x^2\cos x + 2\left[x \cdot \sin x - \int \sin x\,dx\right]$

$-x^2\cos x + 2x\sin x - 2(-\cos x) + C$

$\boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$

ex. $\int sec^3 x\, dx = \int \sec^2 x \cdot \sec x \, dx$

$u = \sec x \quad du = \sec x \tan x \, dx$

$dv = \sec^2 x \quad v = \int \sec^2 x$

$v = \tan x$

$uv - \int v \, du$

$\sec x \tan x - \int \tan x \cdot \sec x \tan x \, dx$

$\sec x \tan x - \int \tan^2 x \cdot \sec x \, dx$

$\sec x \tan x - \int (\sec^2 x - 1) \sec x \, dx$

$\sec x \tan x - \int \sec^3 x - \sec x \, dx$

$\int \sec^3 x \, dx = \sec x \tan x - \cancel{\int \sec^3 x \, dx} + \int \sec x \, dx$

$+ \int \sec^3 x \, dx \qquad + \cancel{\int \sec^3 x \, dx}$

$2 \int \sec^3 x \, dx = \sec x \tan x + \int \sec x \, dx$

$\dfrac{\cancel{2} \int \sec^3 x \, dx}{\cancel{2}} = \dfrac{\sec x \tan x + \ln|\sec x + \tan x| + c}{2}$

$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + c$

ex. $\displaystyle \int \frac{xe^x}{(x+1)^2} dx$

$u = xe^x \quad du = (1e^x + xe^x) dx$

$dv = (x+1)^{-2} \quad v = \int (\overset{u}{x+1})^{-2} \overset{du}{dx} = \int u^{-2} du$

$u = x+1$
$du = dx$

$= -\frac{1}{u} = -\frac{1}{x+1} = v$

$\int \underbrace{xe^x} \left(\frac{1}{(x+1)^2} \right) dx$

$uv - \int v \, du$

$xe^x \left(\frac{-1}{x+1} \right) - \int \left(\frac{-1}{\cancel{x+1}} \right) \overbrace{(e^x + xe^x)}^{e^x \cancel{(x+1)}} dx$

$-\dfrac{xe^x}{x+1} + \int e^x \, dx$

$\boxed{-\dfrac{xe^x}{x+1} + e^x \dfrac{x+1}{x+1} + c}$

$\dfrac{\cancel{-xe^x} \cancel{+xe^x} + e^x}{x+1} = \boxed{\dfrac{e^x}{x+1} + c}$

Assignment
Princeton Book
p 377 1-8 all