AP Calculus AB Sec 4.1 Antiderivatives and Indefinite Integration.

In this chapter we look at the inverse of the derivative called the antiderivative.

ex. $f'(x) = 3x^2$ from experience we know that $f(x) = x^3 + 2$

One small problem, remember that the derivative of a constant is 0....

When writing antiderivative we write it as $f(x) = x^3 + C$

Notation

new word: integration (antiderivative)

$$y = \underbrace{\int f(x)}_{\boxed{\text{integral}}} \underbrace{dx}_{\boxed{\text{variable for integration}}} = F(x) + \underbrace{C}_{\boxed{\text{constant}}}$$

read as "the antiderivative of f with respect to x"

Basic Integration Rules

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad where\, n \neq -1$$

ex $\int x^4\, dx = \frac{x^5}{5} + C$ or $\frac{1}{5}x^5 + C$

BASIC INTEGRATION RULES

Differentiation Formula	*Integration Formula*
$\frac{d}{dx}[C] = 0$	$\int 0\, dx = C$
$\frac{d}{dx}[kx] = k$	$\int k\, dx = kx + C$
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\, dx = k\int f(x)\, dx$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C,\quad n \neq -1$ Power Rule
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$

ex. $\int 4x\,dx$ $= 4\int x^1\,dx = \cancel{4}^{2}\left(\frac{x^2}{\cancel{2}}\right) + c$

$= 2x^2 + C$

ex. $\int (x^2 - 2x + 3)\,dx$

$\int x^2\,dx - 2\int x\,dx + \int 3\,dx$

$\frac{x^3}{3} - \cancel{2}\frac{x^2}{\cancel{2}} + 3x + c$

$\frac{1}{3}x^3 - x^2 + 3x + C$

ex. $\int \frac{1}{x^3}\,dx$ $= \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$

ex. $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \dfrac{x^{3/2}}{3/2} + C = \frac{2}{3}x^{3/2} + C$

ex. $\int \dfrac{3x^2-4}{x^2}\,dx = \int\left(\dfrac{3x^2}{x^2} - \dfrac{4}{x^2}\right)dx$

$\int (3 - 4x^{-2})\,dx$

$\int 3\,dx - 4\int x^{-2}\,dx$

$3x - 4\left(\dfrac{x^{-1}}{-1}\right) + C$

$3x + \dfrac{4}{x} + C$