AP Calculus BC Sec 7.2 Cross Sections

The Disc Method uses a cross section of a circle to find volume.

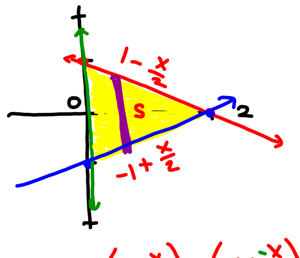
We can use other shapes as cross sections:

General formula: $V = \int_a^b A(x)\,dx$ $\perp$ to xaxis

$V = \int_c^d A(y)\,dy$ $\perp$ to yaxis

ex. Triangular Cross Section

Find the volume of solid whose base is the area bounded by $\boxed{f(x) = 1 - \frac{x}{2}}$, $\boxed{g(x) = -1 + \frac{x}{2}}$ and $\boxed{x = 0}$ and the cross sections $\perp$ to xaxis are $\boxed{\text{equilateral } \Delta\text{s.}}$



$S = (1 - \frac{x}{2}) + (+1 - \frac{x}{2})$

$S = 2 - x$

$A = \frac{1}{2}bh$

$A = \frac{s^2\sqrt{3}}{4}$

$V = \int_a^b A(x)\,dx$

$V = \int_0^2 \frac{(2-x)^2\sqrt{3}}{4}\,dx$

$V = -\frac{\sqrt{3}}{4}\int_0^2 (2-x)^2 \, -dx$ $u = 2 - x$, $du = -1\,dx$

$V = -\frac{\sqrt{3}}{4}\left[\frac{u^3}{3}\right]_0^2$

$V = -\frac{\sqrt{3}}{4}\left[\frac{(2-x)^3}{3}\right]_0^2$

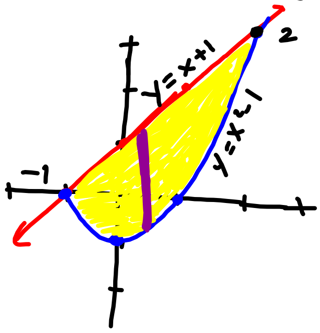
$V = -\frac{\sqrt{3}}{12}\left[\cancel{(2-2)^3} - (2-0)^3\right]$

$V = -\frac{\sqrt{3}}{\cancel{12}_3}(\cancel{-8}^{2}) = \frac{2\sqrt{3}}{3}$

ex Find volume given $\boxed{f(x) = x+1}$, $\boxed{g(x) = x^2-1}$ ⊥ to x axis

Square cross sections

$A = S^2$



$$V = \int_{-1}^{2} (-x^2+x+2)^2\,dx$$

$$V = \int_{-1}^{2} (x^4-2x^3-3x^2+4x+4)\,dx$$

$S = (x+1)+(-x^2+1)$

$S = -x^2+x+2$

$(-x^2+x+2)^2 = -x^2+x+2$
$\quad -x^2+x+2$
$\quad -2x^2+2x+4$
$\quad -x^3+x^2+2x \quad 0$
$\quad x^4-x^3-2x^2 \quad 0 \quad 0$
$\quad x^4-2x^3-3x^2+4x+4$

$$V = \frac{x^5}{5} - \frac{2x^4}{4} - \frac{3x^3}{3} + \frac{4x^2}{2} + 4x \Big]_{-1}^{2}$$

$$V = \frac{x^5}{5} - \frac{x^4}{2} - x^3 + 2x^2 + 4x \Big]_{-1}^{2}$$

$$V = \left(\frac{32}{5} - \cancel{\frac{16}{2}} - \cancel{8} + \cancel{8} + \cancel{8}\right) - \left(\frac{-1}{5} - \frac{1}{4} + 1 + 2 - 4\right)$$

$$V = \left(\frac{32}{5}\right) - \left(-\frac{9}{20} - 1\right) = \boxed{\frac{157}{20} \text{ or } 7.85}$$