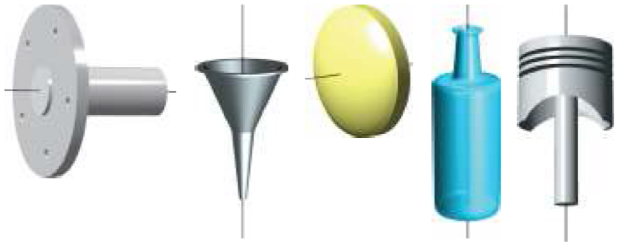
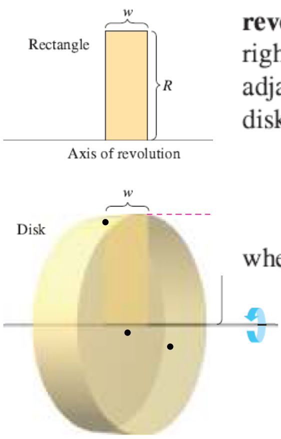
AP Calculus AB Sec 7.2 Volume: The Disc Method

Another important application is finding the volume of a 3-D solid.

Examples in the real world: axles, funnels, pills, bottles and pistons.

Can you think of others?



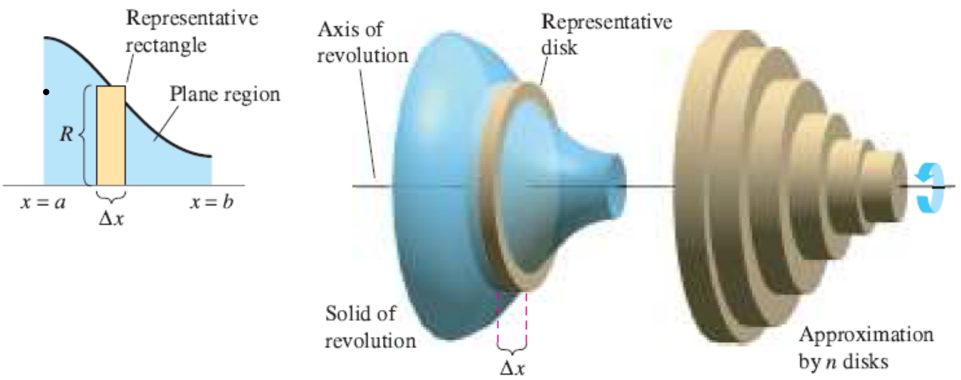


If a region in the plane is revolved about a line, the resulting solid is a **solid of revolution,** and the line is called the **axis of revolution.** The simplest such solid is a right circular cylinder or **disk,** which is formed by revolving a rectangle about an axis adjacent to one side of the rectangle, as shown in Figure 7.13. The volume of such a disk is

$$\text{Volume of disk} = (\text{area of disk})(\text{width of disk})$$
$$= \pi R^2 w$$

where R is the radius of the disk and w is the width.

$$V = \pi \int_a^b [R(x)]^2 \, dx$$



$$V = \pi \int_a^b [R(x)]^2 \, dx$$

ex. Find the volume of solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{3x - x^2}$ and the x-axis ($0 < x \le 3$) about the x-axis.

$$V = \pi \int_a^b [R(x)]^2 dx$$

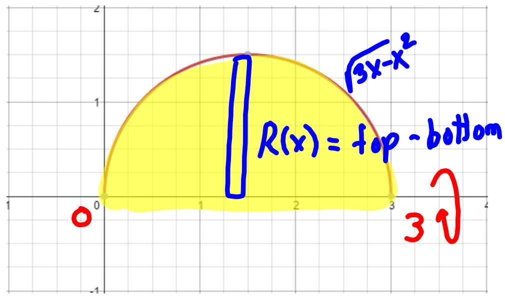
$$V = \pi \int_0^3 \left[\sqrt{3x - x^2}\right]^2 dx$$

$$V = \pi \int_0^3 (3x - x^2) dx$$

$$V = \pi \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$

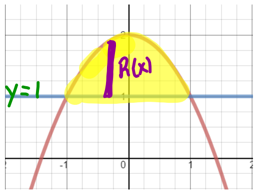
$$V = \pi \left[\left(\frac{27}{2} - \frac{27}{3}\right) - 0\right]$$

$$\pi \left[\frac{81}{6} - \frac{54}{6}\right] = \frac{27}{6}\pi = \frac{9\pi}{2}$$



$R(x) = \left[\sqrt{3x - x^2} - 0\right]$

ex. Find the volume of the solid formed by revolving the region bounded by $f(x) = 2 - x^2$ and $g(x) = 1$ about $y = 1$



① Find points of intersection

$$2 - x^2 = 1$$
$$-x^2 = -1$$
$$x^2 = 1$$
$$x = \pm 1$$

② find $R(x) = \text{top} - \text{bottom}$

$$2 - x^2 - 1 = 1 - x^2$$

③ Write the equation

$$V = \pi \int_{-1}^{1} [1 - x^2]^2 \, dx$$

$$V = \pi \int_{-1}^{1} (1 - 2x^2 + x^4) \, dx$$

$$V = \pi \left[x - \frac{2x^3}{3} + \frac{x^5}{5} \right]_{-1}^{1}$$

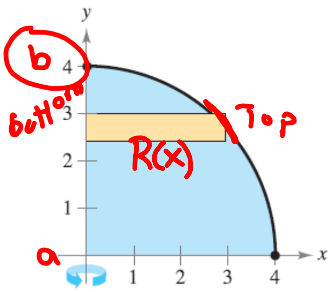
$$V = \pi \left[\left(1 - \frac{2}{3} + \frac{1}{5}\right) + \left(+1 + \frac{-2}{3} + \frac{1}{5}\right) \right]$$

$$\pi \left(2 - \frac{4}{3} + \frac{2}{5} \right)$$

$$\pi \left(\frac{30}{15} - \frac{20}{15} + \frac{6}{15} \right) = \frac{16\pi}{15}$$

Finding volume with respect to the y-axis

8. $y = \sqrt{16 - x^2}$ Rewrite in terms of y (solve for x)



$$y = \sqrt{16 - x^2}$$
$$y^2 = 16 - x^2$$
$$y^2 - 16 = -x^2$$
$$16 - y^2 = x^2$$
$$\sqrt{16 - y^2} = x$$

$$\pi \int_0^4 \left(\sqrt{16 - y^2} \right)^2 dx$$

$$\pi \int_0^4 (16 - y^2) \, dx = \pi \left[16 - \frac{y^3}{3} \right]_0^4$$

$$\pi \left[64 - \frac{64}{3} \right]$$

$$\pi \left[\frac{192}{3} - \frac{64}{3} \right] = \frac{128\pi}{3}$$

Math and alcohol don't mix.
Don't drink and derive.