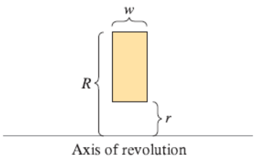
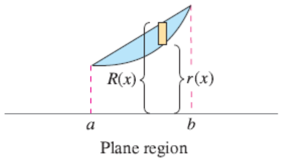
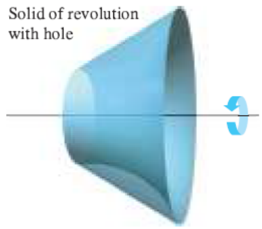
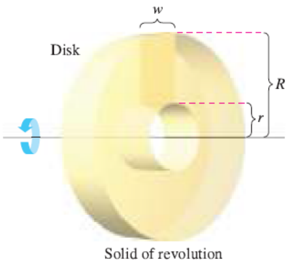
The Washer Method

When your disc has a hole in it, use the washer method.



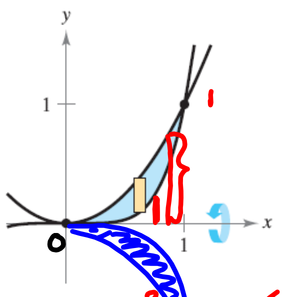






$$V = \pi\int_a^b \left(\underbrace{[R(x)]^2}_{\text{outside}} - \underbrace{[r(x)]^2}_{\text{inside}}\right)\,dx.$$

5. $y = x^2,\ y = x^5$



$R(x) = x^2 \rightarrow (x^2 - 0)$

$r(x) = x^5 \rightarrow (x^5 - 0)$

Washer Method

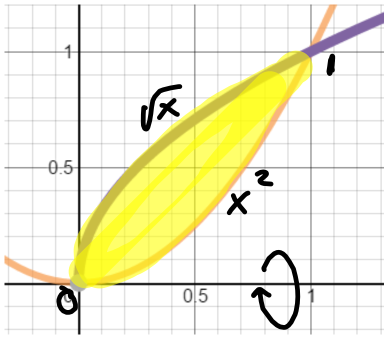
$$\pi\int_a^b [R(x)]^2 - [r(x)]^2\,dx$$

$$\pi\int_0^1 [x^2]^2 - [x^5]^2\,dx$$

$$\pi\int_0^1 x^4 - x^{10}\,dx = \pi\left[\frac{x^5}{5} - \frac{x^{11}}{11}\right]_0^1$$

$$\pi\left[\frac{1}{5} - \frac{1}{11}\right] = \frac{6\pi}{55}$$

ex. Find volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x-axis.



$R(x) = \sqrt{x}$ $\quad r(x) = x^2$

$$V = \pi \int_0^1 \left[(\sqrt{x})^2 - (x^2)^2 \right] dx$$

$$V = \pi \int_0^1 (x - x^4)\, dx$$

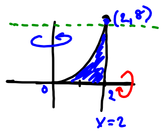
$$V = \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1$$

$$V = \pi \left[\left(\frac{1}{2} - \frac{1}{5} \right) - (0) \right] = \frac{3\pi}{10}$$

12. $y = 2x^2$, $y = 0$, $x = 2$

(a) the y-axis (b) the x-axis

(c) the line $y = 8$ (d) the line $x = 2$



a) washer

$y = 2x^2$

$\frac{1}{2}y = x^2$

$\sqrt{\frac{1}{2}y} = x$

$R(x) = 2$

$r(x) = \sqrt{\frac{1}{2}y}$

$$V = \pi \int_0^8 (2)^2 - \left(\sqrt{\tfrac{1}{2}y}\right)^2 dy$$

$$V = \pi \int_0^8 \left(4 - \tfrac{1}{2}y\right) dy$$

$$V = \pi \left[4y - \tfrac{1}{4}y^2 \right]_0^8$$

$$V = \pi [32 - 16] = 16\pi$$

b) disc method

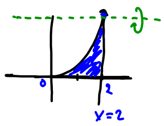
$R(x) = 2x^2$

$$V = \pi \int_0^2 [2x^2]^2 dx$$

$$= \pi \int_0^2 4x^4\, dx$$

$$= \pi \left[\frac{4x^5}{5} \right]_0^2 = \pi \left(\frac{128}{5} \right) = \frac{128\pi}{5}$$

c) washer



$R(x) = 0 \quad r(x) = 2x^2$

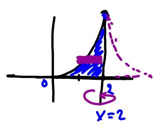
$$\pi \int_0^2 0^2 - [2x^2]^2 dx$$

$$\pi \int_0^2 0 - 4x^4\, dx$$

$$\pi \int_0^2 -4x^4\, dx = -4\pi \left[\frac{x^5}{5} \right]_0^2$$

$$= -4\pi \left[\frac{32}{5} \right] = \boxed{\frac{128\pi}{5}}$$

d) disc $\quad y = 2x^2 \Rightarrow x = \sqrt{\tfrac{1}{2}y}$



$$\pi \int_0^8 \left(2 - \sqrt{\tfrac{1}{2}y}\right)^2 dy$$

$$\pi \int_0^8 \left(4 - 2\sqrt{\tfrac{1}{2}y} + \tfrac{1}{2}y\right) dy$$

$$\pi \left[4y - 2\left(\tfrac{2}{3}\left(\tfrac{1}{2}y\right)^{3/2}\right) + \tfrac{1}{4}y^2 \right]_0^8$$

$$\pi \left[\left(32 - \tfrac{4}{3}(8) + 16 \right) - (0) \right] = \frac{112\pi}{3}$$