

Slope Fields

Sometimes solving a differential equation analytically can be close to impossible. We can use the graphs of the slopes to help sketch the original graph, $f(x)$.

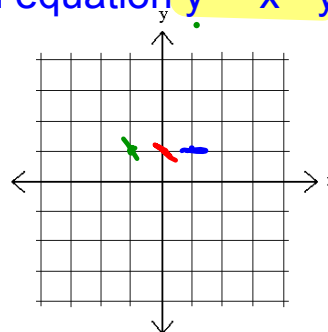
ex. Sketch a slope field for the differential equation $y' = x - y$ for the points $(-1, 1)$, $(0, 1)$ and $(1, 1)$

$$y' = x - y$$

$$y' = -1 - 1 = -2 = \frac{-2}{1} \text{ rise over run}$$

$$y' = 0 - 1 = -1$$

$$y' = 1 - 1 = 0$$



Sketch the slope field for $\frac{dy}{dx} = xy$

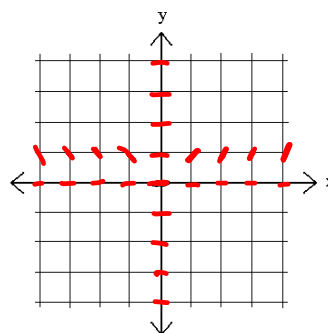
$$(0,0) \rightarrow 0 \cdot 0 = 0$$

$$(1,0) \rightarrow 1 \cdot 0 = 0$$

$$(0,1) \rightarrow 0 \cdot 1 = 0$$

$$(1,1) \rightarrow 1 \cdot 1 = 1$$

$$(2,1) \rightarrow 2 \cdot 1 = 2$$



Match each slope field with its differential equation.

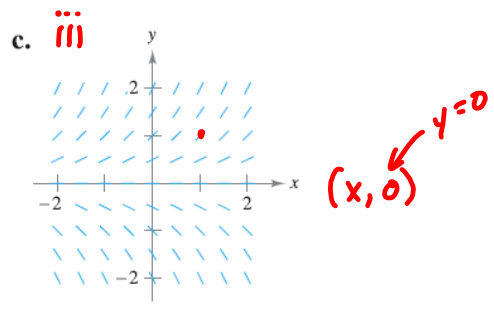
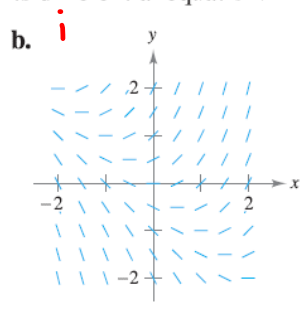
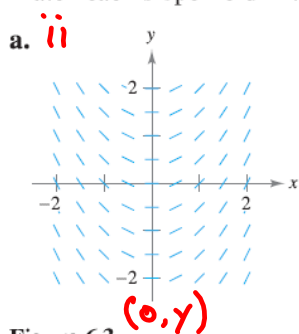


Figure 6.3

i. $y' = x + y$

$y' = x + 0 = x$
 $y' = 0 + y = y$
 $(1, -1) \quad y = 1 + -1 < 0$

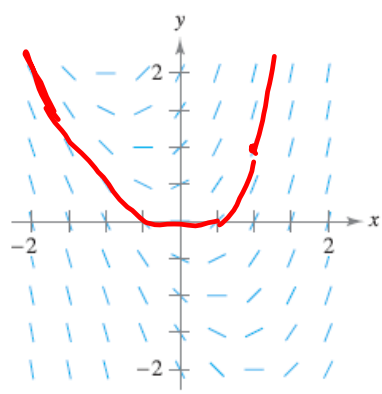
ii. $y' = x$

$y' = x$
 $y' = 0$

iii. $y' = y$

$y' = 0$
 $y' = 1 \quad (1, 1)$
 $y' = 2 \quad (2, 2)$
 $y' = 3 \quad (3, 3)$

Graph a particular solution through the point (1, 1)



Slope field for $y' = 2x + y$

ex. a) Sketch the slope field for the differential equation, b) use the slope field to sketch the solution that passes through the given point.

$$y' = y + xy, (0, -4)$$

| | | | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|---|---|----|---|---|
| x | -5 | -5 | -5 | -4 | -4 | -4 | -3 | -3 | -3 | -2 | -2 | -2 | -1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 1 |
| y | -1 | 0 | 1 | -1 | 0 | 1 | -1 | 0 | 1 | -1 | 0 | 1 | -1 | 0 | 1 | -1 | 0 | 1 | -1 | 0 | 1 |
| y' | 4 | 0 | -4 | 3 | 0 | -3 | 2 | 0 | -2 | 1 | 0 | -1 | 0 | 0 | 0 | -1 | 0 | 1 | -2 | 0 | 2 |

