AP Calculus - Sec 1.3 Evaluating Limits Analytically

Basic Limit Rules

1. $\lim\limits_{x \to c} b = b$ $\qquad \lim\limits_{x\to 2} 3 = 3$

2. $\lim\limits_{x \to c} x = c$ $\qquad \lim\limits_{x\to -4} x = -4$

3. $\lim\limits_{x \to c} x^n = c^n$ $\qquad \lim\limits_{x\to 3} x^2 = 3^2 = 9$

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Other Properties

1. $\lim\limits_{x \to c} [b\, f(x)] = b\,\mathcal{L}$ $\qquad \lim\limits_{x\to 1} 3x^2 = 3 \lim\limits_{x\to 1} x^2$

2. $\lim\limits_{x \to c} [f(x) \pm g(x)] = \mathcal{L} \pm \mathcal{K}$ $\qquad \lim\limits_{x\to 2}(3x^2 + 4x) = \lim\limits_{x\to 2} 3x^2 + \lim\limits_{x\to 2} 4x$

3. $\lim\limits_{x \to c} [f(x)g(x)] = \mathcal{L}\mathcal{K}$ $\qquad \lim\limits_{x\to 1}\left[(3x^2+4x)(2x-5)\right] = \lim\limits_{x\to 1}(3x^2+4x) \cdot \lim\limits_{x\to 1}(2x-5)$

4. $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\mathcal{L}}{\mathcal{K}}$ $\qquad \lim\limits_{x\to -2} \dfrac{3x^2}{x+7} = \dfrac{\lim\limits_{x\to -2} 3x^2}{\lim\limits_{x\to -2} x+7}$

5. $\lim\limits_{x \to c} [f(x)]^n = \mathcal{L}^n$ $\qquad \lim\limits_{x\to 4}\left(3x^4 - 5\right)^6 = \left(\lim\limits_{x\to 4}(3x^4-5)\right)^6$

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First process with limits is direct substitution:

ex. $\lim_{x \to 3} 3x^2 = 3(3)^2 = 27$

ex. $\lim_{x \to -2} \frac{2x^3}{3x+2} = \frac{2(-2)^3}{3(-2)+2} = \frac{-16}{-4} = 4$

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When direct substitution fails, next step is to try factoring/simplifying

ex. $\lim_{x \to -1} \frac{2x^2 - x - 3}{x+1} = \frac{2(-1)^2 - (-1) - 3}{(-1)+1} = \frac{0}{0} = \text{undef.}$

-6

$-3 \quad 2$

$\lim_{x \to -1} \frac{(2x-3)\cancel{(2x+2)}}{\cancel{x+1}} = \lim_{x \to -1} 2x-3 = 2(-1)-3 = -5$

$x+1$

Fake factoring

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If factoring doesn't work and a square root is involved with a fraction. The next method to try is rationalizing the square root.

ex. $\lim_{x\to 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$ mult. by conjugate

$\lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$ foil top / don't distribute bottom

$\lim_{x\to 3} \frac{\cancel{x-3}\ 1}{\cancel{(x-3)}(\sqrt{x+1}+2)}$ cancel top → bottom

$\lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$ substitute

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Find the limit by finding the lowest common denominator.

The function isn't factorable, no square roots to rationalize. Next step, get LCD and simplify to one fraction.

ex. $\lim_{x\to 0} \frac{\frac{1}{x+6} - \frac{1}{6}}{x}$ get common denominator, add

$\lim_{x\to 0} \frac{\frac{6-(x+6)}{(x+6)6}}{\frac{x}{1}} \cdot \frac{1}{x}$

$\lim_{x\to 0} \frac{\cancel{6}-x-\cancel{6}}{6(x+6)\cancel{x}} = \lim_{x\to 0} \frac{-1}{6(x+6)} = \frac{-1}{6(6)} = \frac{-1}{36}$

Jun 29-9:11 PM

The Squeeze Theorem - proof is in textbook, basically it helps us find special trigonometric limits.

Memorize these!

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

ex. $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x}}{\frac{x}{1}} \cdot \frac{1}{x}$

$\lim_{x \to 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right) = \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1}{\cos x} = 1 \cdot 1 = 1$

ex. $\lim_{x \to 0} \frac{\sin 4x}{x}$

$\lim_{x \to 0} \frac{4}{4} \left(\frac{\sin 4x}{x} \right) = 4 \left(\lim_{x \to 0} \frac{\sin \cancel{4x}^{\,y}}{\cancel{4x}_{\,y}} \right) = 4(1) = 4$

Note:
let $y = 4x$
if $x \to 0, y = 0$