Sec 4.5 Integration by Substitution

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du = F(u) + c$$

ex $\int (x^2+1)^2 (2x)\, dx$ Let $u = x^2+1$

$du = 2x\, dx$

$\int u^2\, du$

$\frac{1}{3}u^3 + c = \boxed{\frac{1}{3}(x^2+1)^3 + c}$

Jan 25-7:23 AM

$\frac{1}{2}\int 2x\, (x^2+1)^2\, dx$ Let $u = x^2+1$

$du = 2x\, dx$

$2 \cdot \frac{1}{2} = 1$

$\frac{1}{2}\int u^2\, du$

$\frac{1}{2}\left(\frac{u^3}{3}\right) + c$

$\boxed{\frac{1}{6}(x^2+1)^3 + c}$

ICP p.306
11, 13, 15

$\int u^{1/2}\, du$

$\frac{2}{3}u^{3/2}$

Jan 25-7:55 AM

Change of variable

evaluate $\frac{1}{2}\int_0^1 2x\left(x^2+1\right)^3 dx$

Let $u = x^2+1$

$du = 2x\,dx$

$x = 0 \quad u = 1$

$x = 1 \quad u = 2$

$\frac{1}{2}\int_{\cancel{0}\,1}^{\cancel{1}\,2} u^3\,du$

$\frac{1}{2}\left[\frac{u^4}{4}\right]_1^2 = \frac{1}{2}\left[\frac{16}{4} - \frac{1}{4}\right]$

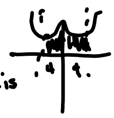
$\frac{1}{2}\left[\frac{15}{4}\right] = \frac{15}{8}$

Jan 25-4:39 PM

Integration of even and odd functions

1. If f is an even function $f(x) = f(-x)$ symmetric to y-axis

   then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

   $f(x) = x^4 + x^2 + 3$

   all exponents are even, constants allowed

2. If f is an odd function $f(x) = -f(x)$ symmetric to origin

   then $\int_{-a}^{a} f(x)\,dx = 0$

   $f(x) = 4x^3 + 2x$ all exponents are odd, no constants allowed

ex. $\int_{-2}^{2} (x^5 - 4x^3 + 6x)\,dx = 0$

↳ odd function

Jan 25-4:43 PM