AP Calculus: Sec 5.7 Inverse Trig Functions: Integration and Completing the Square

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + c$$

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ex. $\frac{1}{3}\int \frac{3dx}{\cancel{2}+\cancel{9x^2}}$

$\sqrt{2}^2 \quad (3x)^2$

THEOREM 5.17 INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x, and let $a > 0$.

1. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$
2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
3. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

$a = \sqrt{2}$

$u = (3x)$

$du = 3\,dx$

$$\frac{1}{3}\int \frac{du}{a^2+u^2}$$

$\sqrt{a^2} = \sqrt{2}$

$a = \sqrt{2}$

$$= \frac{1}{3}\left[\frac{1}{a}\arctan\frac{u}{a}\right] + c$$

$$= \frac{1}{3}\left[\frac{1}{\sqrt{2}}\arctan\frac{3x}{\sqrt{2}}\right] + c$$

$$= \frac{1}{3\sqrt{2}}\arctan\frac{3x}{\sqrt{2}} + c$$

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ex. $\int \frac{dx}{\sqrt{e^{2x}-1}}$

$\int \frac{dx}{\sqrt{(e^x)^2-(1)^2}}$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

$= \frac{1}{1}\operatorname{arcsec}\frac{e^x}{1} + C$

$= \operatorname{arcsec} e^x + C$

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$u = e^x$

$du = e^x dx$

$\frac{du}{e^x} = dx$

$\frac{du}{u} = dx$

$a = 1$

$e^{2x} = (e^x)^2$ or $(e^2)^x$

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ex $\int \frac{x+2}{\sqrt{4-x^2}}\,dx$

$-\frac{1}{2}\int \frac{-2x}{\sqrt{4-x^2}}\,dx + 2\int \frac{2}{\sqrt{4-x^2}}\,dx$

$(2)^2 \quad (x)^2$

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2. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
3. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

$u = 4-x^2$

$du = -2x$

$a = 2$

$u = x$

$du = 1\,dx$

$-\frac{1}{2}\int \frac{du}{\sqrt{u}} + 2\int \frac{du}{\sqrt{a^2-u^2}}$

$-\frac{1}{2}\int u^{-1/2}\,du + 2\arcsin\frac{x}{2} + C$

$-\frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{1}u^{1/2}\right] +$

$-\sqrt{4-x^2} + 2\arcsin\frac{x}{2} + C$

Homework

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1, 3, 5

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$4x + 5y - 2z = -14$

$7x - y + 2z = 42$

$3x + y + 4z = 28$

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rref([A])

$$\left[\begin{array}{ccc|c} 4 & 5 & -2 & -14 \\ 7 & -1 & 2 & 42 \\ 3 & 1 & 4 & 28 \end{array}\right] = \left[\begin{array}{cccc} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 5 \end{array}\right]$$

1

0

0

rref

$x = 4$

$y = -4$

$z = 5$

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5. $\displaystyle\int \frac{1}{x\sqrt{4x^2-1}}\,dx$

$\displaystyle\int \frac{2\,dx}{2x\sqrt{(2x)^2-(1)^2}}$

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3. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

$a = 1$

$u = 2x$

$du = 2\,dx$

$= \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

$= \frac{1}{1}\operatorname{arcsec}\frac{|2x|}{1} + C$

$= \operatorname{arcsec}|2x| + C$

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2. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
3. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

3. $7\int \frac{7}{16 + x^2}\,dx$

$4^2 + (x)^2$

$a = 4$

$u = x$

$du = 1\,dx$

$= 7\left(\frac{1}{a} \arctan \frac{u}{a} + C\right)$

$= \frac{7}{4} \arctan \frac{x}{4} + C$

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ex. $\int \frac{3x^3 - 2}{x^2 + 4}\,dx$

$$\begin{array}{r} 3x \\ x^2+4 \enclose{longdiv}{3x^3 - 2} \\ -3x^3 - 12x \\ \hline -12x - 2 \end{array}$$

$\int 3x\,dx + \int \frac{-12x - 2}{x^2 + 4}\,dx$

$\int 3x\,dx + \int \frac{-12x}{x^2+4}\,dx + \int \frac{-2}{x^2+4}\,dx$

$\int 3x\,dx - \frac{12}{2}\int \frac{2x}{x^2+4}\,dx - 2\int \frac{1}{x^2+4}\,dx$

$u = x^2 + 4$

$du = 2x\,dx$

$a = 2$

$u = x$

$du = 1\,dx$

$\frac{3x^2}{2} - 6\int \frac{1}{u}\,du - 2\int \frac{1}{2^2 + x^2}\,dx$

$\frac{3}{2}x^2 - 6\ln|x^2+4| - \cancel{2}\left(\cancel{\tfrac{1}{2}} \arctan \frac{x}{2}\right) + C$

$\frac{3}{2}x^2 - 6\ln(x^2+4) - \arctan \frac{x}{2} + C$

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ex. $\displaystyle\int \frac{dx}{x^2-4x+7}$

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3. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$

complete the square

$(x^2-4x+4)+7-4$

$(x-2)^2+(\sqrt{3})^2$

$$\int \frac{dx}{\underbrace{(\sqrt{3})^2}_{a}+\underbrace{(x-2)^2}_{u}}$$

$$= \frac{1}{\sqrt{3}}\arctan\frac{x-2}{\sqrt{3}} + C$$

assignment
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