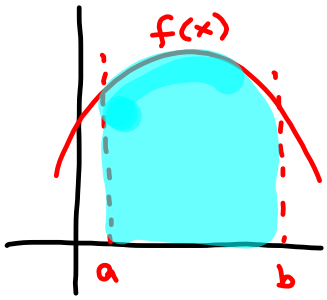
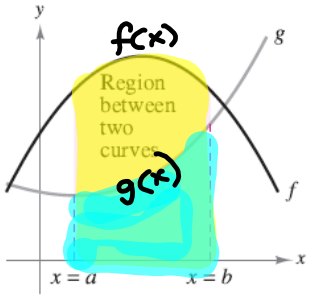
AP Calculus AB Sec 7.1 Area of a Region Between Two Curves

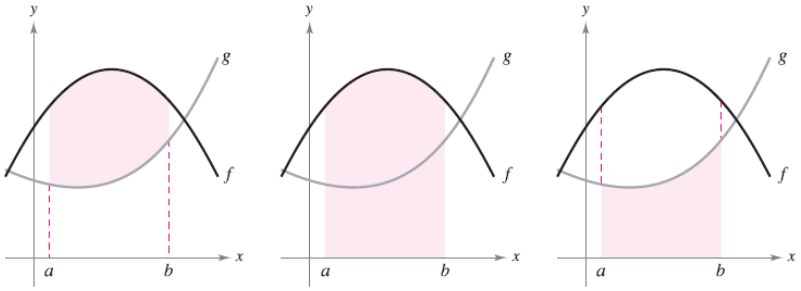


$$A = \int_a^b f(x)\,dx$$



$$A = \int_a^b [f(x) - g(x)]\,dx$$

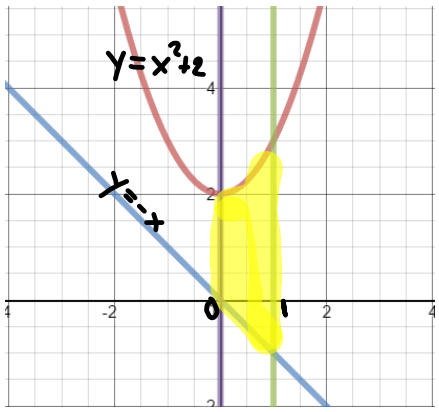
Another visual representation of the area between two curves



Area of region between f and g	=	Area of region under f	−	Area of region under g
$\int_a^b [f(x) - g(x)]\,dx$	=	$\int_a^b f(x)\,dx$	−	$\int_a^b g(x)\,dx$

ex. Find the area of the region bounded by the graphs of

$y = x^2 + 2$, $y = -x$, $x = 0$, $x = 1$



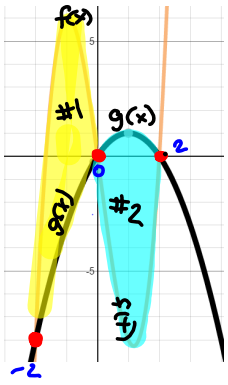
$$\int_0^1 \left[(x^2+2) - (-x)\right] dx$$

$$\int_0^1 x^2 + x + 2 \, dx$$

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big]_0^1 = \left(\frac{1}{3} + \frac{1}{2} + 2\right) - (0)$$

$$= \frac{17}{6}$$

ex. Curves that intersect at more than two points: Find the area of the region between the graphs of:



$f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$

① find points of intersection

$$3x^3 - x^2 - 10x = -x^2 + 2x$$

$$-2x \qquad -2x$$

$$3x^3 - 12x = 0$$

$$3x(x^2 - 4) = 0$$

$3x = 0$ $x^2 - 4 = 0$

$x = 0$ $x = \pm 2$

② set up integrals

$$\int_{-2}^{0} 3x^3 - x^2 - 10x - (-x^2 + 2x)\,dx + \int_0^2 -x^2 + 2x - (3x^3 + x^2 - 10x)\,dx$$

$$\int_{-2}^{0} 3x^3 - 12x \, dx + \int_0^2 -3x^3 + 12x \, dx$$

$$\frac{3x^4}{4} - \frac{12x^2}{2}\Big]_{-2}^{0} + \frac{-3x^4}{4} + \frac{12x^2}{2}\Big]_0^2$$

$$((0) - (12 - 24)) + ((-12 + 24) - (0))$$

$$12 + (12) = 24$$

assignment:

Princeton Book

p. 348 1-4 all