Sec 8.7 Indeterminate Forms and L'Hopital's rule

Indeterminate Forms come from taking the limit of a rational function and getting an answer like one of the following:

$$\frac{0}{0} \ or \ \frac{\infty}{\infty}$$

In the past section we divided to "simplify" the expression to be able to use the rule:

$$\lim_{x\to\infty}\frac{1}{x}=0 \quad or \quad \lim_{x\to 0}\frac{1}{x}=\infty$$

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"Simplifying" doesn't always work, so here is another way to find the limit of a rational function. Only to be used if you have an indeterminate form.

L'Hopital's Rule: $\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$

example from last section:

$$\lim_{x\to\infty}\frac{2x-1}{x+1}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{2}{1}=2$$

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Indeterminate form of $\frac{0}{0}$

$$\lim_{x\to 0}\frac{e^{2x}-1}{x} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{2e^{2\cancel{x}^{0}}}{1} = \frac{2}{1} = 2$$

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Indeterminate form of $\frac{\infty}{\infty}$

$$\lim_{x\to\infty}\frac{ln\cancel{x}^{\infty}}{\cancel{x}^{\infty}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{1}{x}}{1} = \lim_{x\to\infty}\frac{1}{x} = 0$$

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What if L'Hopital's rule doesn't work the first time? Try, try again :)

$$\lim_{x\to-\infty}\frac{x^2}{e^{-x}}=\frac{\infty}{\infty}$$

$$\lim_{x\to-\infty}\frac{2x}{-e^{-x}}=\frac{\infty}{\infty}$$

$$\lim_{x\to-\infty}\frac{2}{e^{-x}}=0$$

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Indeterminate form $0\bullet\infty$

$$\lim_{x\to\infty}e^{-x}\sqrt{x}=$$

$$\lim_{x\to\infty}e^{-x}\cdot\lim_{x\to\infty}\sqrt{x}=0\cdot\infty=0$$

$$\lim_{x\to\infty}\frac{\sqrt{x}}{e^{x}}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{1}{2\sqrt{x}}}{\frac{e^{x}}{1}}\cdot\frac{1}{e^{x}}=\lim_{x\to\infty}\frac{1}{2\sqrt{x}e^{x}}=0$$

$$\frac{1}{2\sqrt{\infty}e^{\infty}}$$

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Indeterminate form 1^{∞}

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = 1^{\infty}$$

$$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$$

$$\ln y = \frac{x}{1}\ln \lim_{x\to\infty}\left(1+\frac{1}{x}\right)$$

$$\ln y = \lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$$

$$\ln y = \lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}}$$

$$\ln y = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}}$$

$$\ln y = \frac{1}{1+0}$$

$$\cancel{e^{\ln}} y = e^{1}$$

$$y = e \longrightarrow \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e$$

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Indeterminate form of 0^0

$$\lim_{x\to 0^+}(\sin x)^{x} = 0^{0}$$

$$y = \lim_{x\to 0^+}(\sin x)^{x}$$

$$\ln y = \frac{x}{1}\ln \lim_{x\to 0^+}(\sin x)$$

$$\ln y = \lim_{x\to 0^+}\frac{\ln \sin x}{\frac{1}{x}} = \frac{0}{0}$$

$$\ln y = \lim_{x\to 0^+}\frac{\frac{1}{\sin x}\cos x}{-\frac{1}{x^2}}$$

Note: $\frac{\cos x}{\sin x} = \cot x$

$\cot x = \frac{1}{\tan x}$

$$\ln y = \lim_{x\to 0^+}\frac{\cot x}{-\frac{1}{x^2}} = \frac{0}{0}$$

$$\ln y = \lim_{x\to 0^+}\frac{-x^2}{\tan x}$$

$$\ln y = \lim_{x\to 0^+}\frac{-2x}{\sec^2 x} = \frac{0}{1} = 0$$

$$\cancel{e^{\ln}} y = e^{0}$$

$$y = e^{0} \Rightarrow \boxed{y=1}$$

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Indeterminate form of ∞ - ∞

$$\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) = \infty - \infty$$

$$\lim_{x\to 1^+}\left(\frac{x-1-\ln x}{(\ln x)(x-1)}\right) = \frac{0}{0}$$

$$\lim_{x\to 1^+}\left(\frac{1-0-\frac{1}{x}}{(\ln x)(1)+\left(\frac{1}{x}\right)(x-1)}\right)$$ product rule

$$\lim_{x\to 1^+}\left(\frac{1-\frac{1}{x}}{\frac{x\ln x}{x}+\frac{x-1}{x}}\right)$$

$$\lim_{x\to 1^+}\left(\frac{\frac{x-1}{\cancel{x}}}{\frac{x\ln x+x-1}{\cancel{x}}}\right)$$

$$\lim_{x\to 1^+}\left(\frac{x-1}{x\ln x+x-1}\right) = \frac{0}{0}$$

$$\lim_{x\to 1^+}\left(\frac{1}{\cancel{x}\left(\frac{1}{\cancel{x}}\right)+1\cdot\ln x+1}\right)$$

$$\lim_{x\to 1^+}\left(\frac{1}{2+\ln x}\right) = \boxed{\frac{1}{2}}$$

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Final Note:

L'Hopital's Rule can only be used with the Indeterminate Forms!

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