AP Calculus Sec 4.1 day 2

Finding a particular solution

ex. Find the general solution of equation $f'(x) = \frac{1}{x^2}$ and find the particular solution that satisfies the initial condition F(1) = 0

$\int f'(x)\,dx = F(x) + C$

$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$ ⟹ $F(x) = -\frac{1}{x} + 1$

$F(1) = -\frac{1}{1} + C = 0$

$-1 + C = 0$

$C = 1$

ex. Find the general solution of equation $f'(x) = 3x^2$ and find the particular solution that satisfies the initial condition $F(2) = 3$

$\int 3x^2\,dx = 3\frac{x^3}{3} + C$

general solution $F(x) = x^3 + C$

$F(2) = (2)^3 + C = 3$

$8 + C = 3$

$C = -5$

Particular Solution

$F(x) = x^3 - 5$

With a partner:

ex. Find the general solution of equation $f'(x) = 10x - 12x^2$ and find the particular solution that satisfies the initial condition $F(3) = 2$

$$\int 10x - 12x^2\,dx$$

$$F(x) = \frac{10x^2}{2} - \frac{12x^3}{3} + C$$

$$F(x) = 5x^2 - 4x^3 + C$$

$$F(3) = 5(3)^2 - 4(3)^3 + C = 2$$

$$45 - 108 + C = 2$$

$$-63 + C = 2$$

$$C = 65$$

$$F(x) = 5x^2 - 4x^3$$

An Application involving gravity $s(t) = -16t^2 + v_0t + s_0$

p 254 **EXAMPLE 8 Solving a Vertical Motion Problem**

A ball is thrown upward with an initial velocity of 64 feet per second from an initial height of 80 feet. $s'(t) = v(0) = 64$ $s(0) = 80$

a. Find the position function giving the height s as a function of the time t.

b. When does the ball hit the ground?

If you were not given $s(t) = -16t^2 + v_0t + s_0$ this is how you would derive the position function

$v'(t) = a(t) = -32$ ft/sec² known for gravity

$$v(t) = \int -32\,dt$$

$$v(t) = -32t + C$$

$$v(0) = -32(0) + C = 64 \quad C = 64 \text{ given } v_0$$

$s'(t) = v(t) = -32t + 64$ new equation with gravity and velocity

$$s(t) = \int (-32t + 64)\,dt$$

$$s(t) = \frac{-32t^2}{2} + 64t + C$$

$$s(t) = -16t^2 + 64t + C$$

$$s(0) = -16(0)^2 + 64(0) + C = 80 \quad C = 80 \text{ given, initial height}$$

80

derived position function

a) $s(t) = -16t^2 + 64t + 80 = 0$

$-16(t^2 - 4t - 5) = 0$

$-16(t - 5)(t + 1) = 0$

$t = 5$, ~~$t = -1$~~

Sec 4.1 day 2 homework. Textbook page 256 #57-65 odd, 71, 75

In Exercises 57–64, solve the differential equation.

57. $f'(x) = 6x,\ f(0) = 8$

58. $g'(x) = 6x^2,\ g(0) = -1$

59. $h'(t) = 8t^3 + 5,\ h(1) = -4$

60. $f'(s) = 10s - 12s^3,\ f(3) = 2$

61. $f''(x) = 2,\ f'(2) = 5,\ f(2) = 10$

62. $f''(x) = x^2,\ f'(0) = 8,\ f(0) = 4$

63. $f''(x) = x^{-3/2},\ f'(4) = 2,\ f(0) = 0$

65. ***Tree Growth*** An evergreen nursery usually sells a certain type of shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by $dh/dt = 1.5t + 5$, where t is the time in years and h is the height in centimeters. The seedlings are 12 centimeters tall when planted ($t = 0$).

(a) Find the height after t years.

Vertical Motion **In Exercises 71–74, use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)**

71. A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go?

Vertical Motion **In Exercises 75–78, use $a(t) = -9.8$ meters per second per second as the acceleration due to gravity. (Neglect air resistance.)**

75. Show that the height above the ground of an object thrown upward from a point s_0 meters above the ground with an initial velocity of v_0 meters per second is given by the function

$$f(t) = -4.9t^2 + v_0t + s_0.$$