# Sec 5.2 Logarithmic Functions + Integration

$$\boxed{\int \frac{1}{x}\,dx = \ln|x| + c}$$

ex. $2\int \frac{2}{x}\,dx = 2\int \frac{1}{x}\,dx = 2\ln|x| + c$

ex. $\frac{1}{2}\int \frac{1}{2x-1}2\,dx$

Let $u = 2x-1$

$du = 2\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|2x-1| + c$

ex. $\int \frac{3x^2+1}{x^3+x}\,dx$

Let $u = x^3+x$

$du = 3x^2+1\,dx$

$\int \frac{1}{u}\,du = \ln|x^3+x| + c$

ex. $\int \frac{x^2-8}{x}\,dx = \int\left(\frac{x^2}{x} - \frac{8}{x}\right)dx$

$= \int\left(x - \frac{8}{x}\right)dx$

$= \int x\,dx - 8\int \frac{1}{x}\,dx$

$$\boxed{\frac{x^2}{2} - 8\ln|x| + c}$$

Assignment p 340 1-11 odd

Day 2

More integrals examples

ex. $\int \frac{x^2+x+1}{x^2+1}\,dx$

since degrees are the same in numerator and denominator we need to divide to "break up" equation.

$$\begin{array}{rl} & 1 + \frac{x}{x^2+1} \\ x^2+1 \big) & \cancel{x^2} + x + \cancel{1} \\ & -\cancel{x^2} + \quad \cancel{-1} \\ \hline & \quad\quad x \end{array}$$

$\int \left(1 + \frac{x}{x^2+1}\right) dx$

$\int 1\,dx + \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$

use substitution

$u = x^2+1$

$du = 2x\,dx$

$\int 1\,dx + \frac{1}{2}\int \frac{1}{u}\,du$

$x + \frac{1}{2}\ln|u| + c$

$x + \frac{1}{2}\ln|x^2+1| + C$

ex. $\int \frac{2x}{(x+1)^2}\,dx$ Let $u = x+1 \xrightarrow{\text{solve for } x} u-1=x$

$du = 1\,dx$

$2\int \frac{x}{(x+1)^2}\,dx$

break fraction apart

$$2\int \frac{u-1}{u^2}\,du = 2\int\left(\frac{u}{u^2}-\frac{1}{u^2}\right)du$$

$$= 2\int \frac{1}{u}\,du - 2\int u^{-2}\,du$$

$$= 2\ln|u| - 2\,\frac{u^{-1}}{-1} + c$$

$$= 2\ln|x+1| + \frac{2}{x+1} + c$$

ex. $\int \frac{1}{x\ln x}\,dx$ break apart to $\int \frac{1}{x}\cdot\frac{1}{\ln x}\,dx$ Let $u = \ln x$

$du = \frac{1}{x}dx$

$= \int \frac{1}{u}\,du$ ← substitute back

$= \ln|u| + C$ integrate

$= \ln|\ln x| + C$ put back the substitute ex. $\int \frac{1}{\sqrt{x}+1}\,dx$

Let $u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

$$2\int \frac{1}{u+1}(2u\,du)$$

$$2\int \frac{u}{u+1}\,du$$

now divide to break apart

$$\begin{array}{r} 1+\frac{-1}{u+1} \\ u+1\,\overline{)\,u\phantom{+1}} \\ -\,u+1 \\ \hline -1 \end{array}$$

$$2\int\left(1-\frac{1}{u+1}\right)du$$

break apart

$$2\int 1\,du - 2\int \frac{1}{u+1}\,du$$

integrate

$$2u - 2\ln|u+1| + C$$

substitute $\sqrt{x}$ back for u

$$2\sqrt{x} - 2\ln|\sqrt{x}+1| + C$$

ex. Finding area integrate like before

$\frac{1}{2}\int_0^3 \frac{2x}{x^2+1}\,dx$

Let $u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}\int_0^3 \frac{1}{u}\,du$

$\frac{1}{2}\left[\ln|x^2+1|\right]_0^3$

$\frac{1}{2}\left[\ln|3^2+1| - \ln|0^2+1|\right]$

$\frac{1}{2}\left[\ln 10 - \ln 1\right]$

$\frac{1}{2}\left[\ln 10 - 0\right] = \frac{1}{2}\ln 10 = \boxed{1.151}$

ex evaluate $\int \tan x \, dx$

$-\int -\frac{\sin x}{\cos x} \, dx$ } Trig identity

Let $u = \cos x$

$du = -\sin x \, dx$

$-\int \frac{1}{u} \, du$

$-\ln|u| + c$

$-\ln|\cos x| + c$

Last example!

evaluate $\int \sec x \, dx$

$\int \sec x \left( \frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$

multiplied by 1 in form of: $\frac{\sec x + \tan x}{\sec x + \tan x}$

why? Look at u substitution

$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$

Let $u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x \, dx$

$\int \frac{du}{u}$

$= \ln|u| + C$

$= \ln|\sec x + \tan x| + C$

This example is insane best is to memorize all integrals in tan box on page 339

Assignment: page 340 13-25 odd, 31-39 odd

You have the next 2 days (Thurs & Fri)

to work on these problems in class.

Worth 10 points, due on Tuesday.