Sec 8.7 Indeterminate Forms and L'Hopital's rule

Indeterminate Forms come from taking the limit of a rational function and getting an answer like one of the following:

$$\frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

In the past section we divided to "simplify" the expression to be able to use the rule:

$$\lim_{x\to\infty}\frac{1}{x}=0 \quad or \quad \lim_{x\to 0}\frac{1}{x}=\infty$$

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"Simplifying" doesn't always work, so here is another way to find the limit of a rational function. Only to be used if you have an indeterminate form.

L'Hopital's Rule: $\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$

example from last section:

$$\lim_{x\to\infty}\frac{2x-1}{x+1}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{2}{1}=2$$

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Indeterminate form of $\frac{0}{0}$

$$\lim_{x\to 0}\frac{e^{2x}-1}{x}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{2e^{2x}}{1}=\frac{2}{1}=2$$

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Indeterminate form of $\frac{\infty}{\infty}$

$$\lim_{x\to\infty}\frac{\ln x}{x}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{1}{x}}{1}=\lim_{x\to\infty}\frac{1}{x}=0$$

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What if L'Hopital's rule doesn't work the first time? Try, try again :)

$$\lim_{x\to-\infty}\frac{x^2}{e^{-x}}=\frac{\infty}{\infty}$$

$$\lim_{x\to-\infty}\frac{2x}{-e^{-x}}=\frac{\infty}{\infty}$$

$$\lim_{x\to-\infty}\frac{2}{e^{-x}}=0$$

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Indeterminate form $0\cdot\infty$

$$\lim_{x\to\infty}e^{-x}\sqrt{x}=$$

$$\lim_{x\to\infty}e^{-x}\cdot\lim_{x\to\infty}\sqrt{x}=0\cdot\infty\neq 0$$

$$\lim_{x\to\infty}\frac{\sqrt{x}}{e^{x}}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{1}{2\sqrt{x}}}{e^{x}}=\lim_{x\to\infty}\frac{1}{2\sqrt{x}\,e^{x}}=0$$

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Indeterminate form 1^{∞}

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = 1^{\infty}$$

$$y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

$$\ln y = x \ln \lim_{x\to\infty}\left(1+\frac{1}{x}\right)$$

$$\ln y = \lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$$

$$\ln y = \lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}}$$

$$\ln y = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}}$$

$$\ln y = \frac{1}{1+0}$$

$$\cancel{e^{\ln}} y = e^1$$

$$y = e \longrightarrow \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

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Indeterminate form of 0^0

$$\lim_{x\to 0^+}(\sin x)^x = 0^0$$

$$y = \lim_{x\to 0^+}(\sin x)^x$$

$$\ln y = x \ln \lim_{x\to 0^+}(\sin x)$$

$$\ln y = \lim_{x\to 0^+}\frac{\ln \sin x}{\frac{1}{x}} = \frac{0}{0}$$

$$\ln y = \lim_{x\to 0^+}\frac{\frac{1}{\sin x}\cos x}{-\frac{1}{x^2}}$$

Note: $\frac{\cos x}{\sin x} = \cot x$

$\cot x = \frac{1}{\tan x}$

$$\ln y = \lim_{x\to 0^+}\frac{\cot x}{-\frac{1}{x^2}} = \frac{0}{0}$$

$$\ln y = \lim_{x\to 0^+}\frac{-x^2}{\tan x}$$

$$\ln y = \lim_{x\to 0^+}\frac{-2x}{\sec^2 x} = \frac{0}{1} = 0$$

$$\cancel{e^{\ln}} y = e^0$$

$$y = e^0 \Rightarrow \boxed{y=1}$$

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Indeterminate form of ∞ - ∞

$$\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) = \infty - \infty$$

$$\lim_{x\to 1^+}\left(\frac{x-1-\ln x}{(\ln x)(x-1)}\right) = \frac{0}{0}$$

$$\lim_{x\to 1^+}\left(\frac{1-0-\frac{1}{x}}{(\ln x)(1)+\left(\frac{1}{x}\right)(x-1)}\right) \text{ product rule}$$

$$\lim_{x\to 1^+}\left(\frac{1-\frac{1}{x}}{\ln x+\frac{x-1}{x}}\right)$$

$$\lim_{x\to 1^+}\left(\frac{\frac{x-1}{\cancel{x}}}{\frac{x\ln x + x-1}{\cancel{x}}}\right)$$

$$\lim_{x\to 1^+}\left(\frac{x-1}{x\ln x + x - 1}\right) = \frac{0}{0}$$

$$\lim_{x\to 1^+}\left(\frac{1}{x\left(\frac{1}{x}\right)+1\cdot\ln x+1}\right)$$

$$\lim_{x\to 1^+}\left(\frac{1}{2+\ln x}\right) = \boxed{\frac{1}{2}}$$

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Final Note:

L'Hopital's Rule can only be used with the Indeterminate Forms!

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Do the following circled Problems from the AP Calculus Problem Book p165

EVALUATE EACH OF THE FOLLOWING LIMITS.

1299. $\lim_{x\to 0}\frac{e^x}{x}$

1300. $\lim_{x\to\infty}\frac{e^{4x}}{5x}$

1301. $\lim_{x\to\infty}\frac{(x+5)^2}{e^{3x}}$

1302. $\lim_{x\to 0}\frac{e^{3x}-2^x}{3x}$

1303. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

1304. $\lim_{x\to\infty}3x\ln\left(1+\frac{1}{x}\right)$

1305. $\lim_{x\to 3}\frac{x^2-9}{x-3}$

1308. $\lim_{x\to 0}\frac{\sin(8x)}{6x}$

1309. $\lim_{x\to 0}\frac{\tan(3x)}{2x}$

1310. $\lim_{x\to\pi/2}\frac{1-\sin x}{x-\pi/2}$

1311. $\lim_{x\to 1}\frac{3x^2-5x+2}{x-1}$

1312. $\lim_{x\to\infty}\frac{3x^2-5x+2}{x-1}$

1313. $\lim_{x\to 0+}x^x$

1314. $\lim_{x\to\infty}(\ln x)^{1/x}$

1315. $\lim\ (1+2x)^{1/(2\ln x)}$

Oct 5-8:47 AM