

AP Calculus - Sec 1.3 Evaluating Limits Analytically

Basic Limit Rules

1. $\lim_{x \rightarrow c} b = b$

$$\lim_{x \rightarrow 2} 3 = 3$$

2. $\lim_{x \rightarrow c} x = c$

$$\lim_{x \rightarrow -4} x = -4$$

3. $\lim_{x \rightarrow c} x^n = c^n$

$$\lim_{x \rightarrow 3} x^2 = 3^2 = 9$$

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Other Properties

1. $\lim_{x \rightarrow c} [b f(x)] = b \mathcal{L}$

$$\lim_{x \rightarrow 1} 3x^2 = 3 \lim_{x \rightarrow 1} x^2 = 3(1) = 3$$

2. $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \mathcal{L} \pm \mathcal{K}$

$$\lim_{x \rightarrow 2} (3x^2 + 4x) = \lim_{x \rightarrow 2} 3x^2 + \lim_{x \rightarrow 2} 4x$$

3. $\lim_{x \rightarrow c} [f(x)g(x)] = \mathcal{L}\mathcal{K}$

$$\lim_{x \rightarrow 1} [(3x^2 + 4x)(2x - 5)] = \lim_{x \rightarrow 1} (3x^2 + 4x) \cdot \lim_{x \rightarrow 1} (2x - 5)$$

4. $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\mathcal{L}}{\mathcal{K}}$

$$\lim_{x \rightarrow 2} \frac{3x^2}{x+7} = \frac{\lim_{x \rightarrow 2} 3x^2}{\lim_{x \rightarrow 2} x+7}$$

5. $\lim_{x \rightarrow c} [f(x)]^n = \mathcal{L}^n$

$$\lim_{x \rightarrow 4} (3x^4 - 5)^6 = (3(4)^4 - 5)^6 = \text{a big \#}$$

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First process with limits is direct substitution:

$$\text{ex. } \lim_{x \rightarrow 3} 3x^2 = 3(3)^2 = 27$$

$$\text{ex. } \lim_{x \rightarrow -2} \frac{2x^3}{3x+2} = \frac{2(-2)^3}{3(-2)+2} = \frac{2(-8)}{-6+2} = \frac{-16}{-4} = 4$$

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Evaluating Limits Analytically with substitution

When direct substitution fails, next step is to try **factoring/simplifying**

$$\text{ex. } \lim_{x \rightarrow -1} \frac{2x^2 - x - 3}{x+1} = \frac{2(-1)^2 - (-1) - 3}{(-1)+1} = \frac{0}{0}$$

$$\lim_{x \rightarrow -1} \frac{(2x-3)\cancel{(x+1)}}{x+1} = \lim_{x \rightarrow -1} 2x-3 = \frac{2(-1)-3}{-2-3} = -5$$

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$$\text{ex. } \lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x^2 - 9} = \frac{0}{0}$$

$$\lim_{x \rightarrow 3} \frac{(x-4)(x-3)}{(x+3)(x-3)} = \lim_{x \rightarrow 3} \frac{x-4}{x+3} = \frac{3-4}{3+3} = \frac{-1}{6}$$

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$$\text{ex. } \lim_{x \rightarrow -2} \frac{x^2 + 2x}{x + 2} = \frac{0}{0}$$

$$\lim_{x \rightarrow -2} \frac{x(x+2)}{x+2} = \lim_{x \rightarrow -2} x = -2$$

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If factoring doesn't work and a square root is involved with a fraction. The next method to try is rationalizing the square root.

$$\text{ex. } \lim_{x \rightarrow 3} \left(\frac{\sqrt{x+1} - 2}{x-3} \right) \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2}$$

$$\lim_{x \rightarrow 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \frac{1}{4}$$

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Find the limit by finding the lowest common denominator.

The function isn't factorable, no square roots to rationalize. Next step, get LCD and simplify to one fraction.

$$\text{ex. } \lim_{x \rightarrow 0} \frac{\frac{1}{x+6} - \frac{1}{6}}{x} = \lim_{x \rightarrow 0} \frac{\frac{6 - (x+6)}{6(x+6)}}{\frac{x}{1}} \cdot \frac{1}{x} = \lim_{x \rightarrow 0} \frac{-x-1}{6x(x+6)} = \frac{-1}{36}$$

$$\frac{\frac{1}{x} + \frac{1}{y}}{xy} = \frac{y+x}{xy}$$

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{6}} = \frac{5}{6}$$

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The Squeeze Theorem - proof is in textbook, basically it helps us find special trigonometric limits.

Memorize these!

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

and

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

ex. $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x}}{\frac{x}{1}} \cdot \frac{1}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{\cos x} \cdot \frac{1}{x}$

$= \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{\cos x} = \frac{1}{1}$

$= 1$

ex. $\lim_{x \rightarrow 0} \frac{\sin 4x}{x}$

$= 4 \lim_{x \rightarrow 0} \frac{\sin x}{x} = 4$

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