AP Calculus BC

Infinite limit - a limit that increases or decreases without bound.

note: lim f(x) = ∞ does not mean that the limit exists,
but how it fails to exist.

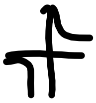
Lets look at the graph of these functions:

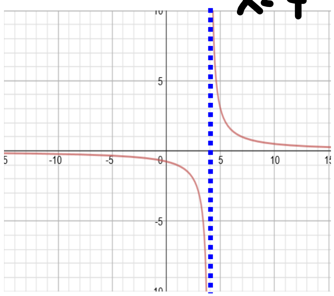
$\lim_{x \to 4} f(x) = \text{DNE}$

$f(x) = \frac{3}{x-4}$ $= 0$

$x = 4$

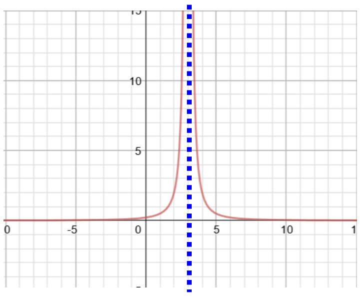
$f(x) = \frac{1}{x}$





$\lim_{x \to 3} f(x) = \infty$

$f(x) = \frac{2}{(x-3)^2}$ $= 0$ $x = 3$



Vertical Asymptotes

If the function is rational (fraction), there will probably be at least one asymptote.

Reminder:
To find a vertical asymptote, set the denominator equal to zero and solve for x.

example: find the vertical asymptote(s):

$f(x) = \frac{1}{2(x+1)}$

VA: $x = -1$

$f(x) = \frac{x^2+1}{x^2-1}$

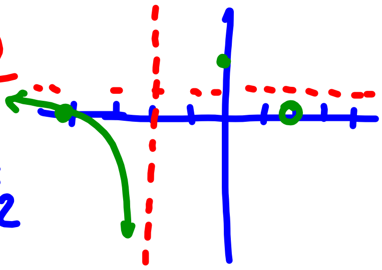
$x = \pm 1$

Before finding the vertical asymptotes should always try to reduce the function.

If when reducing, the denominator cancels with a factor of the numerator, is it still an asymptote?

ex. $f(x)=\dfrac{x^2+2x-8}{x^2-4} = \dfrac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

hole at $x=2$ VA at $x=-2$

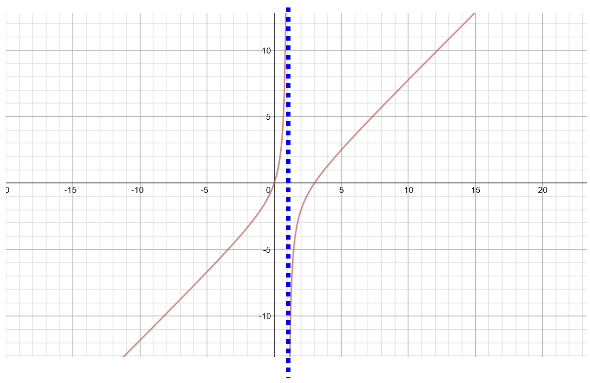


Applying your knowledge of vertical asymptotes find the limits:

$\lim\limits_{x\to 1^-}\dfrac{x^2-3x}{x-1}$ and $\lim\limits_{x\to 1^+}\dfrac{x^2-3x}{x-1} = -\infty$

VA: $x=1$

$\lim\limits_{x\to 1^-} \infty$ or $-\infty$



If $\lim\limits_{x\to c} f(x) = \infty$ and $\lim\limits_{x\to c} g(x) = L$

Rules: $\lim\limits_{x\to c}\left[f(x) \pm g(x)\right] = \infty$ where $L > 0$

$\infty \pm L$

$\lim\limits_{x\to c}\left[f(x)g(x)\right] = \infty$ where $L > 0$

$\infty \cdot L$

$$\lim_{x\to c}\left[\frac{g(x)}{f(x)}\right] = 0$$

$\frac{L}{\infty}$ ← the larger the #, the smaller the fraction

ex. $\lim\limits_{x\to 0^+}\left(x^2 - \frac{1}{x}\right)$

$\lim\limits_{x\to 0^+} x^2 - \lim\limits_{x\to 0^+} \frac{1}{x}$

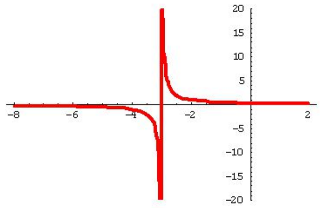
$0 - \infty = -\infty$

Infinite Limits

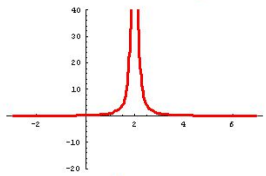
Mathematicians are always looking for patterns. They found one with limits to infinity.

For all $n > 0$,

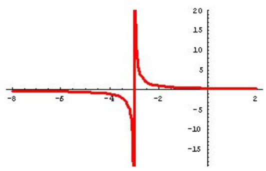
$$\lim_{x\to a^+} \frac{1}{(x-a)^n} = \infty$$



$$\lim_{x\to a^-} \frac{1}{(x-a)^n} = \infty \text{ if } n \text{ is even}$$



$$\lim_{x\to a^-} \frac{1}{(x-a)^n} = -\infty \text{ if } n \text{ is odd}$$



More Graphs