Sec 2.5 Complex Zeros and the Fundamental Theorem of Algebra

Fundamental Theorem of Algebra

A polynomial function of degree n has n complex zeros (real and non real). Some zeros might be repeated.

ex. How many zeros does the following polynomial have?

$$f(x) = x^{4} + 3x^{2} - 1$$

Write polynomial function in standard form and identify the zeros of the function.

a)
$$f(x) = (3x - 2)(x + 4)$$
 factored form

1st: find zeros by setting each () = 0

$$3 \times -2 = 0 \times +4 = 0 \times = -4$$

3x-2=0 x+4=0 $x=\frac{2}{3}$ x=-4 2nd: write in standard form by FOILing

$$(3x-2)(x+4)$$

$$3x^{2}+12x-2x-8 = 3x^{2}+10x-8$$

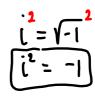
Write polynomial function in standard form and identify the zeros of the function.

b)
$$f(x) = (x - 2i)(x + 2i)$$

1st: find zeros by setting each () = 0

$$X-2i=0 \qquad X+2i=0$$

$$X=2i \qquad X=-2i$$



2nd: write in standard form by FOILing

$$(x-2i)(x+2i)$$

 $x^{2}+\frac{2ix-2ix}{x^{2}+4}-4x^{2}$

What happens when you multiply these?

$$(x + 3i)(x - 3i) = x^{2} - 9i^{2} = x^{2} + 9$$

$$(x - 5i)(x + 5i) = x^{2} + 25$$

$$(x + 4i)(x - 4i) = x^{2} + 16$$

$$(x - i6)(x + i6) = x^{2} + 2$$

$$(x + i6)(x - i6) = x^{2} + 3$$

$$(x - i6)(x + i6) = x^{2} + 3$$

$$(x - i6)(x + i6) = x^{2} + 5$$

c)
$$f(x) = (x - 5)(x - i\sqrt{2})(x + i\sqrt{2})$$

- 1. Find zeros $\chi = 5$, $i\sqrt{2}$, $-i\sqrt{2}$
- 2. Put in standard form

$$(x-5)(x-i\sqrt{2})(x+i\sqrt{2})$$

$$(x-5)(x^2+2)$$

$$x^3+2x-5x^2-10 = x^3-5x^2+2x-10$$

d)
$$f(x) = (x - 3)(x - i)(x + i)$$

- 1. zeros: X = 3, i, -i
- 2. standard form:

$$(x-3)(x-3)(x-i)(x+i)$$

$$(x^{2}-3x-3x+9)(x^{2}+1)$$

$$(x^{2}-6x+9)(x^{2}+1)$$

$$x^{4}+x^{2}-6x^{3}-6x+9x^{2}+9$$

$$x^{4}-6x^{3}+10x^{2}-6x+9$$

$$x^{4}-6x^{3}+10x^{2}-6x+9$$