Sec 2.5 Complex Zeros and the Fundamental Theorem of Algebra

Fundamental Theorem of Algebra

A polynomial function of degree n has n complex zeros (real and non real). Some zeros might be repeated.

ex. How many zeros does the following polynomial have?

$f(x) = x^4 + 3x^2 - 1$

4 roots: rational $\frac{a}{b}$, irrational $\sqrt{\ }$, complex $a+bi$

Write polynomial function in standard form and identify the zeros of the function.

a) $f(x) = (3x - 2)(x + 4)$ factored form

1st: find zeros by setting each ( ) = 0

$3x-2=0$ $x+4=0$

$x=\frac{2}{3}$ $x=-4$

2nd: write in standard form by FOILing

$(3x-2)(x+4)$

$3x^2+12x-2x-8 = 3x^2+10x-8$

Write polynomial function in standard form and identify the zeros of the function.

b) $f(x) = (x - 2i)(x + 2i)$

$i^2 = \sqrt{-1}^2$

$\boxed{i^2 = -1}$

1st: find zeros by setting each ( ) = 0

$x - 2i = 0 \qquad x + 2i = 0$

$\boxed{x = 2i \qquad x = -2i}$

2nd: write in standard form by FOILing

$(x - 2i)(x + 2i)$

$x^2 + 2ix - 2ix - 4i^2$ $\quad (-1)$

$\boxed{x^2 + 4}$

What happens when you multiply these?

$(x + 3i)(x - 3i) = x^2 - 9i^2 = x^2 + 9 \qquad i^2 = -1$

$(x - 5i)(x + 5i) = x^2 + 25$

$(x + 4i)(x - 4i) = x^2 + 16$

$(x - i\sqrt{2})(x + i\sqrt{2}) = x^2 + 2$

$(x + i\sqrt{3})(x - i\sqrt{3}) = x^2 + 3$

$(x - i\sqrt{5})(x + i\sqrt{5}) = x^2 + 5$

c) $f(x) = (x - 5)(x - i\sqrt{2})(x + i\sqrt{2})$

1. Find zeros $x = 5, i\sqrt{2}, -i\sqrt{2}$

2. Put in standard form

$(x-5)(x-i\sqrt{2})(x+i\sqrt{2})$

$(x-5)(x^2+2)$

$x^3+2x-5x^2-10 = x^3-5x^2+2x-10$

d) $f(x) = (x - 3)(x - 3)(x - i)(x + i)$

1. zeros: $x = 3, i, -i$

2. standard form :

$(x-3)(x-3)(x-i)(x+i)$

$(x^2-3x-3x+9)(x^2+1)$

$(x^2-6x+9)(x^2+1)$

$x^4+x^2-6x^3-6x+9x^2+9$

$x^4-6x^3+10x^2-6x+9$

My MathLab
Sec 2.5 day 1